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WEIBULL TABLES FOR
BIO-ASSAYING AND FATIGUE TESTING*

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WEIBULL TABLES FOR BIO-ASSAYING AND FATIGUE TESTING*

Summary

This report presents an acceptance sampling procedure and tables of related sampling inspection plans for the evaluation of lot quality in terms of reliable life (or its complement, quantile life). The Weibull distribution (including the exponential distribution as a special case of the Weibull) is assumed as a statistical model for item lifelength. The evaluation of sample items is by attributes with life testing being truncated at the end of a specified period of time. Tables of factors are also provided from which other sampling inspection plans of desired form can be designed and for use in evaluating the operating characteristics of other specified sampling inspection plans in terms of item reliable life.

Introduction

The sampling inspection tables and procedures discussed in this paper evaluate item life for the lot in terms of reliable life (or its equivalent, quantile life)which is defined as the life beyond which some specified proportion of the items in the lot can be expected to survive (see the appendix of this report). They have been prepared to supplement the Weibull plans and procedures for the evaluation of lot quality in terms of mean life and in terms of hazard rate at some specified life which were presented by the authors and published in the Proceedings of the Seventh National Symposium and the Eighth National Symposium (respectively) on Reliability and Quality Control.[1,2] Readers interested in the application of the Military 105C plans for mean life and hazard rate evaluation will find similar plans for such application in other reports by the authors.[3,4]

This and related material on plans for mean life and for hazard rate has also been published as Department of Defense Technical Reports TR3 and TR4.[5,6]

These papers previously published discuss the Weibull distribution at some length, the underlying assumptions required, the relationship between it and the exponential distribution, and much related material. Also, an extensive discussion of the Weibull distribution as a statistical model for lifelength, together with material on estimating the Weibull parameters can be found in a paper by Kao published in the Proceedings of the Sixth National Symposium.[7] Since this material is readily available, a general discussion of the Weibull distribution will not be repeated in this paper.

However, it may be well to note that the Weibull distribution has three parameters. The first is a scale or characteristic life parameter. For the plans and procedures presented here this parameter is not of concern and need not be known or estimated; the methods are independent of its magnitude. The second is a shape parameter, conventionally symbolized by the letter β. This parameter is quite important for the tables and methods presented in this report; they depend directly on its magnitude. For appropriate application, the magnitude of β must be known or must be assumed to approximate some given value. Such knowledge is usually obtained either directly or indirectly from the analysis of past research and inspection results. The third parameter is a location or threshold parameter, commonly symbolized by the letter γ. For the direct use of the ratios and tables presented here, it is assumed that this parameter has zero value; that there is no initial period of item life that is completely free of any risk of failure. For many applications this will be the case. However, if it is known that γ has some value other than zero, it is very easy to allow for this known value. This point will be discussed in a following section of the report and an illustrative example will be given.

Basic tables of conversion factors (Table 1 and Table 2) for the design of required acceptance plans or the evaluation of specified plans, and comprehensive tables of single-sampling acceptance inspection plans have been computed for an extensive range of β, or shape parameter, values. For the conversion factors, β values of $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, 4, and 5 have been covered. Tables of sampling inspection plans have been provided for the range of β values most commonly encountered with the specific values of $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, and $2\frac{1}{2}$ being included. It should be pointed out that β values of less than 1 apply to products whose hazard rate is relatively high in early life and which decreases with the passage of time. The smaller the value for β, the greater the rate of decrease. Such parameters seem to apply generally to a wide range of electronic components such as resistors and transistors. For a β value of exactly 1, the Weibull distribution is the same as the exponential, the exponential being, in effect, a special case of the Weibull. At this parameter value the hazard rate is constant and independent of the passage of time. For β values greater than 1 the hazard rate is relatively low early in life and increases with the passage of time. The larger the value for β, the greater the rate of increase. This form of hazard rate pattern is typical of products for which failure is due to wear out or fatigue, as may be the case, for example, with ball bearings. Thus it should be obvious that the value for this parameter is critical and must be known for the appropriate application of a sampling inspection plan. This is true also, one should observe, in the case of the exponential; it must be known that the hazard rate is constant ($\beta = 1$) if exponential plans are to be appropriately applied.

For each β value included, factors and sampling plans have been computed for each of three reliability indices (or proportions), namely .5, .90, and .99, selected for use in this study to define reliable life.

It can readily be seen that the reliable life ρ_r is equivalent to the quantile of order (1-r) of a distribution (see Reference 8, p. 181). The reliability index r is the specified survival probability at time x and the reliable life ρ_r is the theoretical lifelength associated with r. For example, for a product if ρ = 1000 hours and r = .90, 90% of the items can be expected to have a life of 1000 hours or longer. Hence if r is chosen to be close to unity, ρ_r will be close to zero. On the other hand, if r can be tolerably small, then ρ_r can be very large indeed. The two trivial cases have been omitted here for r = 1 and 0 whence $\rho_r = 0$ and ∞ respectively. A special case of ρ_r is when $r = \frac{1}{2}$, then it is known as median life ξ.

A notable area of application of the reliable life concept can be found in the anti-friction bearing industry where the rated bearing life for a given application is usually the reliable life with the reliability index r set equal to 90%. A bearing manufacturing firm, for example, lists their bearing capacities based upon LB-10 Life ($\rho_{.90}$) equal to 3000 hours and a speed of 500 rpm. If a life of other than 3000 hours is desired or the actual speed is different from 500 rpm, the load capacity can be appropriately adjusted by using one of the so-called trade-off or acceleration factors similar to those well-known in the electronic component industry. (Unfortunately, this information for major electronic components is still not widely available.)

Another example of application employing the notion of reliable life is found in the area of biological assaying where, for example, the efficiency or potency of a poisonous material (insecticide or herbicide) is characterized by its median lethal dose, LD50[9] which is the theoretical dosage corresponding to the insect's (or plant's) reliable life with r = 50% or more, commonly known as its median life.

There are numerous other applications that can be cited in the areas of fatigue testing of metals or components, sensitivity testing of fuzes or primers, and breakdown voltage of dielectric materials or insulators, to just name a few. For this reason, the examples in this report which demonstrate the use of various tables will not be restricted to any specific area of application, although the report is directed mainly to the areas of fatigue testing and biological assaying.

In the area of fatigue (including failures of most anti-friction bearings) testing where the fatigue life for p % survival[10,11] is exactly equal to the reliable life for $r = p/100$, the Weibull distribution is found useful[12,13]. On the other hand, in the area of biological assaying, although the lognormal distribution was traditionally used[9], the Weibull distribution which can be made to have a shape similar to a lognormal distribution should be equally useful.

The Form of the Acceptance Procedure

The following acceptance inspection procedure has been assumed for the plans and methods covered in this paper:

(a) Select at random a sample of n items from each submitted lot.

(b) Place these sample items on life test for some preassigned test time t.

(c) Determine the number of items that fail prior to the termination of the test (at time t).

(d) Compare the number of items that fail with an acceptance number c specified for the selected plan. If the number failed is equal to or less than the acceptance number, accept the lot; if the number failed exceeds it, reject the lot.

Life length and the test period, t, may be in any appropriate measurable units -- minutes, hours, or stress cycles endured, for example. While only single-sample acceptance plans are included in this report, plans may be designed for double-sampling or multiple sampling if desired through use of the basic conversion ratios provided.

The Basic Conversion Factors

One may note that the above acceptance procedure is of the familiar attribute form. The only modification is that the item quality of interest is life and that testing for life is truncated at some time t. Thus the lot is effectively evaluated in terms of the proportion of items, p', that can be expected to fail before the test truncation time. With the shape parameter of the distribution known or given and with the test time, t, specified, this proportion, p', is a function only of the reliable life for the lot, ρ, and the reliability index, r, of the lot that is to have this reliable life. Hence the operating characteristics (see appendix for derivation) of any specified sampling-inspection plan depend only on t, ρ, and r (given a value for β). So that the ratios and sampling plans will be available for general use, the dimensionless quantity t/ρ has been employed rather than working in terms of specific values for test truncation time and reliable life. In application it will be found quite simple to convert from the ratios to specific values of t and ρ or from specified values for these measures to the equivalent ratio. The proportion r expected to survive beyond the reliable life, ρ, could not be cared for in this convenient manner. It has been necessary to compute separately basic factors and tables of plans for each selected value of r. As previously mentioned, these are $r = .50$, $r = .90$, and $r = .99$.

As a foundation for the reliable life plans included in this report, tables of basic conversion factors have accordingly been computed to show for the Weibull distribution the relationship between p' and the ratio t/ρ.

These factors also may provide a basis for the design of other sampling inspection plans for reliable life using techniques commonly employed with the binomial, hypergeometric, or Poisson distributions to design ordinary attribute plans. Also, the conversion factors may be used to evaluate plans in use or ones that have been specified for use. Examples of such applications will be shown.

These tables of factors will be found at the end of this report as Tables 1, a, b, c, and 2, a, b, c, $r = .50$ for a, $r = .90$ for b, and $r = .99$ for c. For convenience in tabulation and use, the value $(t/\rho) \times 100$ has been employed rather than t/ρ and p' is expressed in percent rather than as a decimal fraction. Table 1 lists values for $(t/\rho) \times 100$ for specified values for $p'(\%)$. Table 2 lists values for $p'(\%)$ for specified values for $(t/\rho) \times 100$. In each case, separate tables have been prepared for each of the selected values for r. These two sets of tables are meant to supplement each other so as to provide convenient conversion either way. One may also note that by the provision of these two supplementary sets, a very considerably wider range of conversion values is provided; the factors in one table are considerably expanded in range in the region where they are compressed in the other table and vice versa. The values selected for $p'(\%)$ and $(t/\rho) \times 100$ from which to convert have been determined by the use of a standard preferred number series. Details of the mathematical steps involved in establishing the $(t/\rho) \times 100$ and p' relationships will be found in the appendix at the end of the report.

Example (1)

A sampling inspection plan is required for the evaluation of production lots of a product in terms of reliable life with reliable life defined as the life beyond which 50% of the items can be expected to survive. A reliable life of 1000 hours is considered acceptable and for lots with this reliable life or longer the probability of acceptance should be high, .95 or more. A reliable life of 400 hours is considered unacceptable and lots with this reliable life or less should have a low probability of acceptance, .05 or less. A test truncation time of 100 hours is to be employed. Experience has indicated the Weibull distribution applies with an expected value for the shape parameter of $1\frac{2}{3}$ and for the location parameter of 0. Thus, $\rho = 1000$ at the AQL for which $P(A) \geqq .95$, $\rho = 400$ at the RQL for which $P(A) \leqq .05$, $r = .50$, $t = 100$, $\beta = 1\frac{2}{3}$, and $\gamma = 0$.

Through the use of Table 2a which contains conversion factors for $r = .50$, values for p' at the AQL and the RQL can be determined. For the values for t and ρ specified,

$$(t/\rho) \times 100 = (100/1000) \times 100 = 10 \qquad \text{(at the AQL)}$$

$$(t/\rho) \times 100 = (100/400) \times 100 = 25 \qquad \text{(at the RQL).}$$

By entering Table 2a with these two values and reading from the column for the shape parameter value, β, of $1\frac{2}{3}$, it is found that at the AQL $p' = 1.48(\%)$ and at the RQL $p' = 6.45(\%)$. These are the respective probabilities of item failure before the end of the 100 hour testing period.

With these two values for p', values for n, the sample size, and c, the acceptance number can be determined through any of the well-known methods ordinarily used in the design of attribute sampling inspection plans. The Poisson-based tables prepared by Cameron[14] will serve well for this example. Through use of these tables it is found that an acceptance number of either 4 or 5 will meet the requirements reasonably well. Through further use of Cameron's tables and with an acceptance number of 5, it is found that a sample size of 164 will provide the required consumer's risk. With this acceptance number and sample size, the tables indicate the probability of acceptance at the acceptable quality level will be between .95 and .975 so that the producer's risk requirement will also be met. An alternative procedure for determining c and n and one that is somewhat more precise is to use the beta probability chart (which is based on the binomial distribution) prepared by Kao.[15]

Example (2)

For another application of sampling inspection in terms of reliable life, a Military Standard Plan has been specified, one with an AQL of 1.5% and Sample Size Code Letter K. For single sampling, the sample size for this plan is 110 items and the acceptance number 4. Reliable life is to be defined as the life beyond which 90% of the items can be expected to survive, or $r = .90$. Testing of sample items is to be truncated at 400 hours. The Weibull distribution can be assumed as a lifelength model with $\beta = 2\frac{1}{2}$ and $\gamma = 0$. The user of this plan would like to know what its operating characteristics are in terms of reliable life and in particular what protection he as the consumer will receive.

To determine these characteristics, the first step is to determine for the n and c specified the p' values associated with selected probabilities of acceptance. These values may be obtained approximately by reading them from the Operating Characteristic curves supplied as a part of the 105 Plans or by use of cumulative tables of the Poisson or binomial distribution. Examination of the operating characteristic curve for the selected plan indicates that at $P(A) = .95$, $p' = 1.8\%$ and at $P(A) = .10$, $p' = 7.3\%$ (approximately, in both cases). A check through use of Poisson tables indicates these values are reasonably close to the right percentages.

The next step is to use these percentages to determine from Table Ib, which gives tables of conversion factors for $r = .90$, the corresponding $(t/\rho) \times 100$ values. With these values and with the value for t specified, only a simple computation is required for the final step necessary to find the desired reliable life values. At the acceptable quality level for which $p' = 1.8(\%)$, by interpolation in the column of factors for $\beta = 2\frac{1}{2}$, a value for $(t/\rho) \times 100$ of 49.4 can be found. With $t = 400$, $(400/\rho) \times 100 = 49.4$ or $\rho = 810$ hours. This, then, is the "acceptable" reliable life; the reliable life for which the probability of acceptance will be high. At the unacceptable quality level for which $p' = 7.3(\%)$, interpolation in Table Ib will give a value of 87.5 will be found for $(t/\rho) \times 100$. Substitution of $t = 400$ gives $(400/\rho) \times 100 = 87.5$ or $\rho = 460$ hours. Thus if the reliable life is 460 hours or less the probability of acceptance will be low, .10 or less.

Under the use of the Sample Size Letter, K, selected for this example, alternatives of double-sampling and multiple sampling are available. If double sampling is employed, for example, the first sample size would be 75 and the second 150. The acceptance number would be 2 for the first sample and the rejection number 8. For the combined samples the acceptance number would be 7 and the rejection number 8, as specified in the 105 Standard. All other usual procedures for double-sampling would be employed. The test time for the first sample would be 400 hours, the same as for single sampling; likewise the test time for the second sample would have to be 400 hours. One may note that a possible reduction by double-sampling in the number of sample items that have to be inspected in the long run can be achieved only by a doubling of the duration of the life-testing time for some lots. Under double (or multiple sampling) employing the same Sample Size Code Letter and AQL the operating characteristics will obviously be closely the same as for single sampling.

Example (3)

Suppose that in another application the requirements and conditions are the same as for Example (2) with the exception that γ, the location or threshold parameter, is equal to 250 hours. As before, at $P(A) = .95$, $p' = 1.8\%$ and the corresponding $(t/\rho) \times 100$ value is 49.4. Likewise, the $(t/\rho) \times 100$ value at $P(A) = .10$ for which $p' = 7.3\%$ is 87.5. However, now ρ must be considered in terms of $\gamma = 0$. A new value t_0, which is $t_0 = t - \gamma = 400-250 = 150$ must be computed and used in working with the factors from the table. At the acceptable quality level now $(t_0/\rho_0) \times 100 = 49.4$ or $(150/\rho_0) \times 100 = 49.4$ which results in a value for ρ_0 of 300 hours for the relative reliable life. This is converted back to absolute or real terms by simply adding the value for γ; thus $\rho = 300+250 = 550$ hours for the acceptable reliable life. At the unacceptable quality level, $(t_0/\rho_0) \times 100 = 87.5$ or $(150/\rho_0) \times 100 = 87.5$ which results in a value for ρ_0 of 170 hours. The real or absolute value for the unacceptable reliable life is 170+ 250 or 420 hours. In any case of use of the sampling plans or basic conversion factors presented in this report, when γ has some value greater than 0, all that must be done is to work in terms of t_0 and ρ_0 where $t_0 = t - \gamma$ and $\rho_0 = \rho - \gamma$. The solution in terms of t_0 or ρ_0 is then converted back to absolute terms by adding the value for γ.

Sampling Inspection Plans

This report also includes twenty-four tables of sampling inspection plans. These tables cover eight values of the shape parameter, β, over the range most frequently encountered in practice. For each β value, tables have been prepared for each of the three values of the reliability index, r, for which the relationship between p' and $(t/\rho) \times 100$ has been determined. These tables, Tables 3a1 through 3c8, will be found at the end of the report.

Each table lists for a range of acceptance numbers, c, the minimum sample size, n, to be employed. A plan (c and n) is available for a variety of $(t/\rho) \times 100$ ratios and for each ratio for acceptance numbers ranging from 0 to 10. The plans have been designed so that if 100 times the ratio between the test-truncation time, t, and the reliable life for the lot, ρ, is equal to the ratio value in the selected column heading, the probability of acceptance, P(A) will be .10 or less. That is, a selected plan assures with 90% confidence or more the rejection of lots for which the $(t/\rho) \times 100$ ratio is equal to or greater than the value shown in the column heading. It has been assumed that in the use of these plans the consumer's risk will be of most importance. For this reason the plans have been cataloged by their $P(A) \leqq .10$ ratios. These ratios (as shown in the column headings) are a common measure of consumer protection and may be regarded in the same way as LTPD (lot tolerance per cent defective) values are regarded in describing the operating characteristics of ordinary attributes and variables acceptance plans.

In addition, for each of the plans the $(t/\rho) \times 100$ ratio has been determined for which the probability of acceptance is .95 or more. Each of these $P(A) \geqq .95$ ratio values will be found enclosed in parentheses immediately under the corresponding sample size number. These ratio values may be regarded in the same way that AQL (acceptable quality level) values are as a measure of the producer's risk. If the item life distribution for a lot is such that its $(t/\rho) \times 100$ ratio is equal to or less than the table heading value, the selected plan assures $P(A) = .95$.

Thus the two ratio values, the one in the column heading and the one in parentheses immediately below the sample size number, describe in broad terms the operating characteristics of each plan. If one or the other of these values is specified for an acceptance inspection application, with additional information, a suitable plan may be selected from the tables. Alternatively, the pair of values may be used to determine in approximate terms the operating characteristics of a plan that has been specified or that is in use and whose values for n and c match reasonably well one of the plans in the tables.

To make these plans available for general use, the binomial distribution and the Poisson distribution were employed in their design. For this reason the size of the lot should be relatively large compared to the size of the sample, just as in the case of most other published tables of attribute inspection sampling plans. If the sample requires taking a substantial portion of the lot, the probability values assigned to the $(t/\rho) \times 100$ ratios will not precisely apply. This point, however, should present little difficulty in practice. Binomial tables prepared by Grubbs[16] were used in the design of all plans using acceptance numbers, c, up to 9 and sample sizes, n, up to 150. The remainder of the plans, those for c = 10 and for sample sizes over 150, were designed by employing the Poisson distribution as an approximation to the binomial. Here use was made of np' values prepared by Cameron.[14] In each case of changing from the binomial to the Poisson distribution, the match in sample sizes was checked. It was found to be close in all cases. Furthermore, the slight differences that were found were on the conservative side; the sample size under the Poisson was slightly larger than the number theoretically required under the binomial assumption.

In addition to making sure the sample size is not so large that it constitutes a substantial portion of the lot, a few other practical points in application should be observed. One is that if specified sample sizes are for practical reasons to be rounded off to the nearest number ending in five or zero (or to the nearest one hundred), this rounding off should be to a number larger than the number given in the table. This will assure the retention of the specified consumer's protection, P(A) = .10 or less.

Another point of practice that should ordinarily be followed is that if a plan is not available for which the $(t/\rho) \times 100$ ratio matches closely the desired ratio, a plan should be selected from the column with the next smaller ratio value. By following this conservative practice a confidence level of 90% or greater will be maintained in assuring that the specific minimum reliable life has been met. On the other hand, if some acceptable quality level must be guaranteed (a ratio or a reliable life for which $P(A) \geqq .95$) and a matching ratio value is not available in the tables, a plan with the next higher value should be used. If this is done, a lot with an acceptable reliable life will have $P(A) \geqq .95$. One should also note that when plans with the desired ratios are not available in the tables, interpolation may be employed between the listed sample sizes to find a new plan that does have more nearly the desired operating characteristics. Finally, it should be mentioned that testing of sample items can be terminated after the acceptable number of failures has been exceeded. The lot is to be rejected and so further testing will have no value unless the sampling inspection data is to be used to provide an estimate of the process average for the product or the vendor. In the latter case, testing should continue for the full period, t.

<u>Example (4)</u>

A sampling inspection plan is required for a product which will accept with a probability of .10 or less ($P(A) \leqq .10$) lots whose reliable life is 400 hours or less. In this case reliable life is defined as the life beyond which 90% ($r = .90$) of the items in the lot will survive. It will also be desirable to be able to assure the producer that if the reliable life is 2,000 hours or more, the probability of acceptance will be high, say .95 or greater. A test period of 200 hours is to be employed. Through past experience with the product it has been established that the Weibull distribution applies for item lifelength, with the value for β, the shape parameter, being approximately 1 and for γ, the threshold parameter being 0.

With these specifications for the sampling plan, 100 times the ratio of the test time, t, to the reliable life, ρ) is (200/400) × 100 or 50 at the unacceptable reliable life of 400 hours for which $P(A) \leqq .10$ has been specified. At the acceptable reliable life of 2,000 hours the $(t/\rho) \times 100$ ratio is (200/2,000) × 100 or 20. An inspection plan meeting these ratio requirements will be found in Table 3b4 which lists plans for $\beta = 1$ and $r = .90$. Any plan in the fifth column (headed 50) will meet the unacceptable reliable life requirement. Of the plans assigned to this column, the last one has a ratio value of 20, the value required at the acceptable reliable life. The plan is thus to use a sample size, n, of 301 and an acceptance number, c, of 10.

Example (5)

A plan has been specified for the acceptance inspection of a product which requires that a sample of 375 items be drawn from the lot and tested for 500 hours. If no more than 7 items fail before the end of the test period, the lot is to be accepted; if more than this number fail, it is to be rejected. Data from past inspection and research indicates a value for the shape parameter, β, of $\frac{2}{3}$ applies with the location parameter, γ, being 0. The user of this plan would like to know its operating characteristics in terms of reliable life, with reliable life being defined as the median life or the life beyond which 50% of the items can be expected to survive.

An answer may be found by inspection of that portion of Table 3a1 which tabulates plans for $\beta = \frac{1}{3}$ and $r = .50$. An examination of this table indicates a plan is tabulated approximating the one to be used, the plan for $c = 7$ and $n = 372$. For this plan the $(t/\rho) \times 100$ value for which $P(A) \leqq .10$

is found (in the corresponding column heading) to be 1.0. By the substitution of the test period specified, 500 hours, for t in this ratio, one obtains $(500/\rho) \times 100 = 1.0$ or $\rho = 50{,}000$ hours. Thus if the reliable life is 50,000 hours or less, the probability of acceptance will be .10 or less. For this plan the ratio value at the acceptable reliable life is .19 (as shown by the number in parentheses under the sample size). By substitution of the real value for t, $(500/\rho) \times 100 = .19$ or $\rho = 263{,}000$ hours. These two values for reliable life describe in a practical way the operating characteristics of the plan that has been specified.

Example (6)

For a sixth example consider a case for which it can be assumed the shape parameter, β, will equal approximately 2 and the threshold parameter, γ, will equal 1200 cycles. A plan is required for which the P(A) = .10 or less if the reliable life is 6,000 cycles or less with reliable life being defined as the life beyond which 99% of the items will survive. A test truncation time of 5,000 cycles seems reasonable and could be used. The user would also like to know the effect of cutting the test time to 3,000 cycles.

Reference will be to Table 3c7 which tabulates plans for $\beta = 2$ and for $r = .99$. The first step is to convert the specified values for t and ρ to relative values in terms of $\gamma = 0$. Thus $t_0 = 5{,}000 - 1{,}200 = 3{,}800$ cycles and $\rho_0 = 6{,}000 - 1{,}200 = 4{,}800$ cycles. The $(t_0/\rho_0) \times 100$ ratio is $(3{,}800/4{,}800) \times 100 = 79$ or approximately 80. Any plan in the column with this ratio heading in the table of plans for $\beta = 2$ and $r = .99$ will meet the rejectable quality level requirements. One possibility is the plan for which $n = 349$ and $c = 0$. This allows the minimum sample size possible.

Consider now the proposal to cut the test time to 3,000 cycles. In this case $t_0 = 3{,}000 - 1{,}200 = 1{,}800$ cycles and $\rho_0 = 6{,}000 - 1{,}200 = 4{,}800$ cycles. The $(t_0/\rho_0) \times 100$ ratio is now $(1{,}800/4{,}800) \times 100 = 38$. The nearest ratio available in the table is 40. In the column with this ratio heading, the best plan available (from the standpoint of sample size) is the one for which $c = 0$ and $n = 1400$. The penalty for reducing the test period is thus to increase the sample size from 349 to 1400.

It might be interesting to compare these two possibilities in terms of the acceptable reliable life, the life for which $P(A) = .95$. For the first one ($n = 349$ and $t = 5{,}000$), the ratio at the AQL is 12 (as shown by the figure in parentheses). Thus $(t_0/\rho_0) \times 100 = 12$ or $(3{,}800/\rho_0) \times 100 = 12$ from which one determines that $\rho_0 = 32{,}000$ cycles. Converted back to absolute terms, $\rho = \rho_0 + \gamma = 32{,}000 + 1{,}200$ or 33,200 cycles. This must be the reliable life if the lot is to have a high probability of acceptance. For the second possibility ($n = 1400$ and $t = 3{,}000$), the ratio at the AQL is 5.9. Thus $(1{,}800/\rho_0) \times 100 = 5.9$ or $\rho_0 = 31{,}000$ cycles. Converted to absolute terms, $\rho = 31{,}000 + 1{,}200$ or 32,200 cycles which is approximately the same requirement as for the first plan. Thus it should make no difference to the producer which plan is used. These last computations illustrate a unique feature of the Weibull plans for life and reliability testing; the ability of a plan to discriminate between good and bad lots depends on the size of the acceptance number rather than on the size of the sample (as is the case for ordinary attribute sampling plans). For any given acceptance number and for the same value for β a nearly constant ratio will be found between the acceptable reliable life and the unacceptable reliable life regardless of the general level of these lives. This will also be approximately the case regardless of the value chosen for the proportion r that must survive.

This point has been fully demonstrated in the author's reports for the Weibull mean life plans[1,3] and the Weibull hazard rate plans.[2,4] Table 3 of Reference 1 (or alternatively Table 4 of Reference 3) gives approximate values for $\mu_{.95}/\mu_{.10}$. These same ratios can be used for the reliable life plans presented here, that is, they can be used as $\rho_{.95}/\rho_{.10}$ values. If both the acceptable reliable life with P(A) = .95 and the unacceptable reliable life with P(A) = .10 are specified, use of this table of values will indicate at once what the acceptance number should be. This information is a very helpful start in designing other details of a plan to meet given needs. In the above application, for example, if an acceptable reliable life of 33,000 cycles had been specified, $\rho_{0(.95)}/\rho_{0(.10)}$ = 31,800/4,800 or 6.7. Reference to the table just described would indicate that for $\beta = 2$ the acceptance number c would have to be 0. On the other hand, if an acceptable reliable life of 15,000 cycles had been specified instead (for which ρ_0 = 12,000 - 1,200 or 10,800), the $\rho_{0(.95)}/\rho_{0(.10)}$ ratio would be 10,800/4,800 or 2.2. Reference to the table would indicate the acceptance number must be 3. Reference again to Table 3c7 of this report would indicate the sample size must accordingly be 1,010 if the test period is to be 6,000 cycles (in which case the t/ρ ratio is 80) or must be 4,050 if the test period is to be 3,000 cycles (in which case the t/ρ ratio is 40).

Appendix

Reliable Life as a Life-quality Criterion

This appendix describes the notion of reliable life or quantile life of complementary order which is used as the life-quality criterion for items subject to the testing procedures given in this report.

For an arbitrary lifelength distribution defined over $\gamma \leq x < \infty$ (γ is the threshold or location parameter) with c.d.f. = $F(x)$ and p.d.f. = $f(x)$, the reliability function = $R(x) = 1-F(x)$ and a reliability index r ($0 < r < 1$), the reliable life ρ_r (see Reference 11) is the solution of x in $R(x) = r$ or,

$$\rho_r = R^{-1}(r) \tag{A1}$$

where R^{-1} is the inverse function of R.

If the lifelength of an item follows a Weibull distribution of the form:

$$F(x) = 1 - \exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right], \quad x \geq \gamma;\ \eta, \beta > 0;$$
$$= 0, \text{ otherwise} \tag{A2}$$

and its p.d.f.,

$$f(x) = \frac{\beta}{\eta}\left(\frac{x-\gamma}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right], \quad x \geq \gamma,\ \eta, \beta > 0,$$
$$= 0, \text{ otherwise} \tag{A3}$$

Then the reliable life of order r will be,

$$\rho_r = \gamma + \eta\,(-\ln r)^{b} \tag{A4}$$

where $b = 1/\beta$. In this report, since γ is assumed to be known, Equations (A2, A3 & A4) are not used. Instead, Equations (A5, A6, & A7) are used.

For γ known, there is no loss of generality by assuming $\gamma = 0$. In this case,

$$F(x) = 1 - \exp[-(x/\eta)^{\beta}] \quad \text{and} \tag{A5}$$

$$f(x) = \frac{\beta}{\eta}(x/\eta)^{\beta-1} \exp[-(x/\eta)^{\beta}] \tag{A6}$$

and the reliable life is,

$$\rho_r = \eta\,(-\ln r)^{b} \tag{A7}$$

Now let the testing time be truncated at t and let p' be the probability of failure of an item prior to t, then combining (A5) and (A7),

$$p' = F(t) = 1 - \exp\{-[t(-\ln r)^{b}/\rho_r]^{\beta}\} \tag{A8}$$

which can be simplified as,

$$p' = 1 - \exp[(t/\rho_r)^{\beta} \ln(r)] \tag{A9}$$

It can be noted now that if the truncation time coincides with ρ_r, one would always (for any $\beta > 0$) have $p' = 1 - \exp[\ln(r)] = 1 - r$, which is to be expected. Also since $0 < r < 1$, $\ln(r)$ will always be negative and finite; thus the Weibull c.d.f. in the form of Equation (A8) or (A9) satisfies the conditions: $F(0) = 0$ and $F(\infty) = 1$ and the c.d.f. is monotonic in t for $\beta > 0$.

The inverse of Equation (A9) gives,

$$t/\rho_r = [\ln(1-p') / \ln(r)]^{b} \tag{A10}$$

Notice that Equation (A6) also asserts that for any $\beta > 0$, $p' = 1-r$ if $t = \rho_r$.

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TABLE 1 - a

Table of Values for $(t/\rho) \times 100$ $\qquad$ $r = .50$

p' (%)	Shape Parameter - β										
	1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010				.014	.130	.496	1.20	2.91	7.04	10.9	17.0
.012				.017	.152	.554	1.32	3.13	7.44	11.5	17.7
.015				.022	.177	.632	1.47	3.42	7.95	12.1	18.5
.020				.028	.220	.745	1.69	3.81	8.63	13.0	19.5
.025			.001	.036	.263	.860	1.90	4.20	9.27	13.8	20.5
.030			.001	.043	.299	.960	2.08	4.52	9.79	14.4	21.2
.040			.001	.058	.372	1.14	2.40	5.06	10.7	15.5	22.5
.050			.002	.072	.441	1.30	2.69	5.54	11.4	16.4	23.5
.065			.003	.094	.536	1.52	3.06	6.15	12.4	17.5	24.8
.080			.004	.115	.623	1.73	3.40	6.68	13.1	18.4	25.8
.10			.005	.144	.742	1.98	3.80	7.31	14.0	19.5	27.0
.12			.007	.173	.849	2.20	4.16	7.86	14.8	20.4	28.0
.15			.010	.216	1.01	2.52	4.65	8.59	15.9	21.6	29.3
.20		.001	.015	.289	1.25	2.99	5.37	9.64	17.3	23.2	31.0
.25		.001	.022	.361	1.47	3.42	6.01	10.5	18.5	24.5	32.5
.30		.002	.028	.433	1.68	3.82	6.58	11.3	19.5	25.6	33.7
.40		.003	.044	.579	2.10	4.54	7.61	12.7	21.3	27.6	35.7
.50		.005	.061	.723	2.49	5.19	8.50	13.9	22.8	29.2	37.3
.65		.009	.091	.941	3.01	6.08	9.70	15.5	24.7	31.1	39.3
.80		.013	.125	1.16	3.53	6.89	10.8	16.8	26.2	32.8	41.0
1.0		.021	.175	1.45	4.18	7.88	12.0	18.4	28.1	34.7	42.9
1.2	.001	.030	.230	1.74	4.78	8.80	13.2	19.8	29.7	36.3	44.5
1.5	.001	.048	.322	2.18	5.66	10.1	14.8	21.6	31.7	38.4	46.5
2.0	.002	.085	.497	2.91	7.04	12.0	17.1	24.3	34.6	41.3	49.3
2.5	.005	.133	.698	3.65	8.35	13.7	19.1	26.6	37.0	43.7	51.6
3.0	.008	.193	.921	4.39	9.61	15.3	21.0	28.6	39.2	45.8	53.5
4.0	.020	.347	1.43	5.89	12.0	18.3	24.3	32.2	42.8	49.3	56.8
5.0	.041	.548	2.01	7.40	14.1	21.0	27.2	35.3	45.8	52.1	59.4
6.5	.091	.940	3.02	9.70	17.4	24.7	31.1	39.3	49.7	55.8	62.7
8.0	.174	1.45	4.17	12.0	20.4	28.1	34.7	42.9	53.0	58.9	65.5
10	.351	2.31	5.92	15.2	24.3	32.3	39.0	47.1	56.8	62.4	68.6
12	.627	3.40	7.92	18.4	28.1	36.3	42.9	50.8	60.2	65.5	71.3
15	1.29	5.50	11.4	23.4	33.7	41.9	48.4	56.0	64.7	69.6	74.8
20	3.34	10.4	18.3	32.2	42.7	50.7	56.7	63.6	71.2	75.3	79.7
25	7.15	17.2	26.7	41.5	51.8	59.0	64.4	70.3	76.8	80.3	83.9
30	13.6	26.5	36.9	51.5	60.8	67.1	71.7	76.7	81.9	84.7	87.6
40	40.0	54.3	63.3	73.7	79.7	83.3	85.9	88.5	91.2	92.7	94.1
50	100	100	100	100	100	100	100	100	100	100	100
65	347	229	186	151	136	128	123	118	113	111	109
80	1250	539	354	232	188	166	152	140	129	124	118

TABLE 1 - b

Table of Values for $(t/\rho) \times 100$ r = .90

p' (%)	Shape Parameter - β										
	1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010			.003	.095	.542	1.54	3.08	6.18	12.4	17.6	24.9
.012			.004	.114	.621	1.71	3.38	6.65	13.1	18.4	25.8
.015			.005	.142	.732	1.96	3.77	7.27	14.0	19.4	27.0
.020			.008	.190	.911	2.33	4.36	8.15	15.3	20.9	28.6
.025		.001	.012	.237	1.08	2.66	4.87	8.92	16.3	22.1	29.9
.030		.001	.015	.285	1.23	2.99	5.34	9.59	17.2	23.1	31.0
.040		.001	.023	.380	1.53	3.53	6.16	10.8	18.8	24.8	32.8
.050		.002	.033	.475	1.80	4.03	6.89	11.8	20.1	26.2	34.3
.065		.004	.048	.617	2.20	4.72	7.85	13.1	21.7	28.0	36.1
.080		.006	.066	.759	2.57	5.35	8.71	14.2	23.1	29.5	37.7
.10		.009	.092	.949	3.04	6.12	9.74	15.5	24.7	31.2	39.4
.12		.013	.121	1.14	3.49	6.82	10.7	16.7	26.1	32.7	40.9
.15		.020	.170	1.42	4.12	7.80	11.9	18.3	27.9	34.5	42.7
.20	.001	.036	.262	1.90	5.12	9.27	13.8	20.5	30.5	37.1	45.3
.25	.001	.056	.365	2.37	6.04	10.6	15.4	22.4	32.6	39.2	47.3
.30	.002	.081	.480	2.85	6.93	11.8	16.9	24.1	34.4	41.1	49.1
.40	.006	.145	.743	3.81	8.62	14.1	19.5	27.1	37.5	44.2	52.0
.50	.011	.226	1.04	4.76	10.2	16.1	21.8	29.5	40.1	46.7	54.4
.65	.024	.383	1.54	6.19	12.4	18.8	24.9	32.9	43.4	49.9	57.3
.80	.044	.581	2.04	7.62	14.5	21.3	27.6	35.7	46.2	52.5	59.8
1.0	.087	.910	2.95	9.54	17.2	24.4	30.9	39.1	49.4	55.6	62.5
1.2	.150	1.31	3.88	11.5	19.7	27.2	33.8	42.0	52.2	58.2	64.8
1.5	.295	2.06	5.43	14.3	23.3	31.2	37.9	46.0	55.8	61.5	67.8
2.0	.705	3.68	8.40	19.2	29.0	37.1	43.8	51.6	60.9	66.2	71.9
2.5	1.39	5.78	11.8	24.0	34.3	42.5	49.0	56.5	65.2	70.0	75.2
3.0	2.42	8.36	15.5	28.9	39.4	47.5	53.8	60.9	68.9	73.3	78.0
4.0	5.82	15.0	24.1	38.7	49.1	56.6	62.2	68.4	75.2	78.9	82.7
5.0	11.5	23.7	34.0	48.7	58.3	64.9	69.8	75.0	80.6	83.5	86.6
6.5	26.0	40.7	50.9	63.8	71.4	76.4	79.9	83.6	87.4	89.4	91.4
8.0	49.6	62.6	70.4	79.1	83.9	86.9	89.0	91.0	93.2	94.3	95.4
10	100	100	100	100	100	100	100	100	100	100	100
12	179	147	134	121	116	112	110	108	106	105	104
15	367	238	192	154	138	130	124	119	114	111	109
20	950	449	308	212	176	157	146	135	125	121	116
25	2030	746	451	273	213	183	165	149	135	129	122
30	3880	1150	623	339	250	208	184	163	144	136	128
40		2350	1,070	485	326	258	220	188	161	148	137
50		4330	1,690	658	410	310	257	213	176	160	146
65		9930	3,150	996	562	397	316	251	199	178	158
80			5,970	1,530	774	513	391	298	227	198	173

TABLE 1 - c

Table of Values for $(t/\rho) \times 100$ $r = .99$

p' (%)	Shape Parameter - β										
	1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010		.010	.100	.995	3.15	6.29	9.98	15.8	25.1	31.6	39.8
.012		.014	.131	1.19	3.61	7.02	10.9	17.0	26.5	33.1	41.3
.015		.022	.182	1.49	4.26	8.02	12.2	18.6	28.3	34.9	43.1
.020	.001	.040	.281	1.99	5.30	9.50	14.1	20.8	30.9	37.6	45.6
.025	.002	.062	.392	2.49	6.26	10.9	15.8	22.8	33.0	39.7	47.8
.030	.003	.089	.516	2.99	7.18	12.2	17.3	24.5	34.9	41.6	49.5
.040	.006	.158	.794	3.98	8.91	14.5	20.0	27.5	38.0	44.7	52.5
.050	.012	.248	1.11	4.98	10.5	16.5	22.3	30.1	40.7	47.2	54.9
.065	.027	.418	1.64	6.47	12.8	19.3	25.4	33.4	44.0	50.4	57.8
.080	.050	.634	2.25	7.96	15.0	21.9	28.2	36.3	46.8	53.1	60.3
.10	.099	.990	3.14	9.95	17.7	25.0	31.5	39.7	50.0	56.2	63.0
.12	.170	1.43	4.12	11.9	20.3	27.9	34.6	42.7	52.9	58.8	65.4
.15	.333	2.23	5.76	14.9	24.0	31.9	38.6	46.7	56.5	62.2	68.4
.20	.788	3.96	8.89	19.9	29.8	38.0	44.6	52.4	61.6	66.8	72.4
.25	1.54	6.19	12.4	24.9	35.2	43.4	49.9	57.3	65.9	70.6	75.7
.30	2.66	8.91	16.3	29.9	40.4	48.4	54.6	61.7	69.58	73.9	78.5
.40	6.35	15.9	25.2	39.9	50.2	57.6	63.2	69.2	75.9	79.5	83.2
.50	12.4	24.8	35.2	49.9	59.3	65.9	70.6	75.7	81.2	84.0	87.0
.65	27.3	42.1	52.2	64.9	72.3	77.1	80.6	84.1	87.8	89.8	91.7
.80	51.0	63.8	71.4	79.9	84.5	87.4	89.4	91.4	93.5	94.6	95.6
1.0	100	100	100	100	100	100	100	100	100	100	100
1.2	173	144	132	120	115	112	110	108	106	105	104
1.5	340	226	184	150	136	128	123	118	113	111	109
2.0	812	404	285	201	169	152	142	132	123	119	115
2.5	1600	635	400	252	200	174	159	145	132	126	120
3.0	2780	919	528	303	230	195	174	156	140	132	125
4.0	6700	1,650	818	406	286	232	202	175	152	142	132
5.0		2,600	1,150	510	339	266	226	192	163	150	139
6.5		4,470	1,730	669	416	313	259	214	177	161	146
8.0		6,880	2,390	830	489	356	288	233	189	170	153
10			3,390	1,050	582	410	324	256	202	180	160
12			4,540	1,270	673	460	357	277	214	189	166
15			6,500	1,620	806	531	402	305	231	201	175
20				2,220	1,020	642	471	346	254	217	186
25				2,860	1,240	748	535	383	274	231	196
30				3,550	1,450	851	596	417	292	244	204
40				5,080	1,900	1060	713	481	325	267	219
50				6,900	2,390	1270	831	544	356	288	233
65					3,270	1630	1,020	642	403	320	253
80					4,500	2100	1,270	762	458	356	276

TABLE 2 - a

Table of Values for p' (%) r = .50

$(t/\rho) \times 100$	Shape Parameter - β										
	1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010	3.17	.691	.149	.007							
.012	3.36	.757	.168	.008							
.015	3.62	.846	.195	.010							
.020	3.97	.975	.237	.014	.001						
.025	4.27	1.09	.275	.017	.001						
.030	4.53	1.19	.310	.021	.001						
.040	4.98	1.38	.376	.028	.002						
.050	5.35	1.54	.436	.035	.003						
.065	5.83	1.75	.518	.045	.004						
.080	6.23	1.94	.596	.055	.005						
.10	6.70	2.17	.691	.069	.007						
.12	7.10	2.37	.779	.083	.009						
.15	7.63	2.65	.904	.104	.012	.001					
.20	8.36	3.05	1.09	.139	.017	.002					
.25	8.98	3.41	1.27	.173	.024	.003					
.30	9.51	3.73	1.43	.202	.030	.004					
.40	10.4	4.29	1.73	.277	.044	.007	.001				
.50	11.2	4.79	2.01	.347	.059	.010	.002				
.65	12.1	5.44	2.39	.450	.084	.015	.003				
.80	12.9	6.01	2.74	.554	.110	.022	.004				
1.0	13.9	6.70	3.17	.691	.149	.032	.007				
1.2	14.7	7.31	3.57	.819	.190	.043	.010	.001			
1.5	15.7	8.14	4.13	1.03	.256	.063	.016	.002			
2.0	17.2	9.34	4.98	1.38	.376	.102	.028	.004			
2.5	18.3	10.4	5.75	1.72	.505	.148	.043	.007			
3.0	19.4	11.3	6.47	2.06	.644	.207	.062	.011			
4.0	21.1	12.9	7.79	2.74	.944	.324	.111	.022	.001		
5.0	22.5	14.4	8.98	3.41	1.27	.469	.173	.039	.003		
6.5	24.3	16.2	10.6	4.41	1.80	.726	.293	.075	.008	.001	
8.0	25.8	17.8	12.1	5.39	2.36	1.02	.443	.125	.015	.003	
10	27.5	19.7	13.9	6.70	3.17	1.48	.691	.219	.032	.007	
12	29.0	21.3	15.5	7.98	4.02	2.00	.994	.346	.059	.015	.001
15	30.8	23.5	17.8	9.88	5.37	2.99	1.55	.605	.124	.035	.006
20	33.3	26.7	21.1	12.9	7.79	4.63	2.74	1.23	.324	.111	.022
25	35.4	29.3	24.1	15.9	10.3	6.45	4.24	2.14	.680	.271	.068
30	37.1	31.6	26.6	18.8	13.0	8.90	6.05	3.36	1.25	.560	.168
40	40.0	35.5	31.4	24.2	18.5	14.0	10.5	6.78	3.22	1.76	.698
50	42.3	38.7	35.4	29.3	24.0	19.6	15.9	11.5	6.65	4.24	2.14
65	45.1	42.8	40.6	36.3	32.3	28.7	25.4	21.0	15.2	11.6	7.73
80	47.4	46.2	45.0	42.6	40.2	38.0	35.8	32.8	28.1	24.7	20.3
100	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0

TABLE 2 - b

Table of Values for p' (%) r = .90

(t/ρ) x 100	Shape Parameter - β										
	1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010	.490	.105	.022	.001							
.020	.614	.149	.036	.002							
.030	.703	.182	.047	.003							
.040	.773	.211	.057	.004							
.050	.833	.236	.066	.005							
.065	.909	.269	.079	.007							
.080	.973	.298	.091	.008							
.10	1.05	.333	.105	.011	.001						
.12	1.11	.365	.119	.013	.001						
.15	1.20	.408	.138	.016	.002						
.20	1.32	.470	.167	.021	.003						
.25	1.42	.526	.194	.026	.004						
.30	1.51	.576	.219	.032	.005						
.40	1.66	.664	.265	.042	.007	.001					
.50	1.79	.743	.308	.053	.009	.001					
.65	1.95	.846	.367	.068	.013	.002					
.80	2.09	.938	.421	.084	.017	.003					
1.0	2.24	1.05	.488	.105	.022	.005	.001				
1.2	2.38	1.15	.551	.126	.029	.007	.001				
1.5	2.57	1.28	.638	.158	.039	.009	.002				
2.0	2.82	1.48	.773	.211	.057	.015	.004				
2.5	3.03	1.65	.897	.263	.077	.022	.006	.001			
3.0	3.22	1.81	1.01	.316	.098	.030	.009	.002			
4.0	3.54	2.09	1.22	.420	.144	.049	.017	.003			
5.0	3.81	2.33	1.42	.526	.194	.071	.026	.006			
6.5	4.15	2.65	1.69	.683	.275	.111	.044	.011	.001		
8.0	4.44	2.94	1.94	.840	.363	.156	.067	.019	.002		
10	4.72	3.28	2.24	1.05	.488	.227	.105	.023	.005	.001	
12	5.07	3.58	2.53	1.26	.621	.307	.151	.052	.009	.002	
15	5.44	4.00	2.94	1.57	.837	.446	.237	.092	.019	.005	.001
20	5.96	4.60	3.54	2.09	1.22	.718	.421	.188	.049	.017	.003
25	6.42	5.13	4.10	2.60	1.65	1.04	.656	.329	.104	.041	.010
30	6.81	5.61	4.61	3.11	2.09	1.40	.944	.518	.190	.085	.026
40	7.47	6.45	5.56	4.13	3.06	2.26	1.67	1.06	.496	.270	.108
50	8.02	7.18	6.42	5.13	4.10	3.26	2.60	1.84	1.04	.657	.329
65	8.72	8.14	7.60	6.62	5.76	5.01	4.35	3.53	2.47	1.86	1.21
80	9.32	8.99	8.68	8.09	7.53	7.01	6.52	5.85	4.88	4.22	3.39
100	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
120	10.6	10.9	11.2	11.9	12.6	13.3	14.1	15.3	17.6	19.6	23.1
150	11.4	12.1	12.9	14.6	16.6	18.7	21.1	25.2	33.4	41.3	55.1
200	12.4	13.8	15.4	19.0	23.3	28.4	34.4	44.9	65.4	81.5	

TABLE 2 - c

Table of Values for p' (%) r = .99

(t/ρ) x 100	Shape Parameter - β										
	1/3	1/2	2/3	1	1 1/3	1 2/3	2	2 1/2	3 1/3	4	5
.010	.047	.010	.002								
.020	.059	.014	.003								
.040	.074	.020	.005								
.080	.093	.028	.009								
.10	.101	.032	.010	.001							
.20	.127	.045	.016	.002							
.40	.159	.064	.025	.004	.001						
.80	.201	.090	.040	.008	.002						
1.0	.216	.101	.047	.010	.002						
1.2	.230	.110	.053	.012	.003						
1.5	.248	.123	.061	.015	.004	.001					
2.0	.273	.142	.074	.020	.005	.001					
2.5	.294	.159	.086	.025	.007	.002					
3.0	.312	.174	.097	.030	.009	.003	.001				
4.0	.344	.201	.118	.040	.014	.005	.002				
5.0	.370	.225	.136	.050	.019	.007	.003				
6.5	.404	.256	.163	.065	.026	.010	.004	.001			
8.0	.433	.284	.187	.080	.035	.015	.006	.002			
10	.465	.318	.217	.101	.047	.022	.010	.003			
12	.495	.347	.245	.121	.060	.029	.014	.005	.001		
15	.533	.388	.285	.151	.080	.043	.023	.009	.002	.001	
20	.586	.448	.344	.201	.118	.069	.040	.018	.005	.002	
25	.631	.502	.399	.251	.158	.100	.063	.031	.010	.004	.001
30	.671	.548	.449	.302	.202	.135	.090	.049	.018	.008	.002
40	.738	.634	.545	.401	.296	.218	.161	.101	.047	.026	.010
50	.795	.709	.632	.502	.399	.316	.251	.177	.099	.063	.031
65	.867	.807	.752	.651	.565	.489	.424	.342	.239	.179	.117
80	.929	.895	.863	.801	.744	.691	.641	.574	.476	.411	.329
100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
120	1.06	1.10	1.13	1.20	1.27	1.35	1.44	1.57	1.83	2.06	2.47
150	1.14	1.22	1.31	1.50	1.71	1.96	2.24	2.73	3.81	4.96	7.35
200	1.26	1.41	1.58	2.00	2.50	3.14	3.94	5.53	9.57	14.9	27.5
250	1.36	1.58	1.83	2.48	3.35	4.52	6.09	9.45	19.2	32.5	62.5
300	1.44	1.73	2.07	2.97	4.26	6.08	8.65	14.5	32.4	55.6	91.3
400	1.58	1.99	2.50	3.94	6.18	9.63	14.9	27.5	64.0	92.4	
500	1.70	2.22	2.90	4.90	8.23	13.7	22.2	43.0	88.3		
650	1.86	2.53	3.44	6.32	11.5	20.4	34.6	66.1			
800	1.99	2.80	3.94	7.73	14.9	27.5	47.4	83.8			
1000	2.14	3.13	4.56	9.56	19.5	37.3	63.4				
1500	2.45	3.82	5.93	14.0	31.1	60.0	89.6				

Table 3a1

Sampling Plans for $\beta = 1/3$, $r = .50$

c	n												
	$(t/\rho) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	100	50	25	10	5.0	2.5	1.0	.50	.25	.10	.05	.025	.010
0	4	5	6	8	10	12	16	20	25	34	42	53	72
1	7 (.05)	8 (.03)	10 (.01)	13 (.01)	16	20	27	34	42	57	72	90	122
2	9 (.32)	11 (.16)	14 (.08)	18 (.03)	22 (.02)	28 (.01)	37	46	58	78	98	123	168
3	12 (.66)	14 (.38)	17 (.21)	23 (.08)	28 (.04)	35 (.02)	47 (.01)	58	73	98	123	156	211
4	14 (1.4)	17 (.68)	21 (.34)	27 (.14)	34 (.07)	42 (.03)	56 (.01)	70 (.01)	87	118	148	187	252
5	17 (1.8)	20 (1.0)	24 (.55)	32 (.21)	39 (.11)	49 (.05)	65 (.02)	81 (.01)	102	137	173	217	293
6	19 (2.7)	23 (1.3)	28 (.70)	36 (.29)	45 (.14)	56 (.07)	74 (.03)	92 (.01)	115 (.01)	157	197	247	332
7	21 (3.5)	26 (1.7)	31 (.90)	41 (.36)	50 (.19)	62 (.09)	82 (.04)	103 (.02)	129 (.01)	176 (.01)	220	276	371
8	24 (4.1)	28 (2.3)	34 (1.1)	45 (.47)	56 (.22)	69 (.11)	91 (.05)	113 (.02)	143 (.01)	194	243	304	410
9	26 (4.9)	31 (2.7)	38 (1.3)	49 (.50)	61 (.25)	75 (.13)	100 (.05)	124 (.03)	158 (.01)	212	266	333	448
10	30 (4.9)	35 (2.7)	43 (1.3)	56 (.50)	68 (.25)	84 (.13)	111 (.05)	138 (.03)	172 (.01)	230	288	361	486

$(t/\rho) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

Table 3a2

Sampling Plans for $\beta = 1/2$, $r = .50$

c	n: (t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	100	50	25	10	5.0	2.5	1.0	.50	.25	.10	.050	.025	.010
0	4 (.03)	5 (.02)	7 (.01)	11	15	21	34	47	67	105	150	212	334
1	7 (.62)	9 (.37)	12 (.20)	19 (.07)	26 (.04)	36 (.02)	57 (.01)	80	113	180	253	357	563
2	9 (2.2)	12 (1.1)	17 (.55)	26 (.22)	36 (.11)	50 (.06)	78 (.02)	110 (.01)	156	245	346	488	770
3	12 (3.6)	16 (1.9)	21 (1.0)	32 (.42)	45 (.21)	63 (.10)	98 (.04)	139 (.02)	196 (.01)	308	434	613	967
4	14 (5.8)	19 (2.8)	26 (1.4)	39 (.59)	54 (.30)	75 (.15)	118 (.06)	167 (.03)	234 (.01)	368	519	733	1160
5	17 (6.9)	22 (3.7)	30 (1.9)	45 (.79)	63 (.39)	87 (.20)	137 (.08)	194 (.04)	272 (.02)	427 (.01)	602	851	1340
6	19 (9.1)	25 (4.7)	34 (2.3)	51 (1.0)	71 (.49)	99 (.25)	157 (.09)	220 (.05)	309 (.02)	485 (.01)	684	966	1530
7	21 (11)	28 (5.6)	38 (2.8)	58 (1.1)	80 (.56)	111 (.29)	176 (.11)	246 (.05)	345 (.03)	542 (.01)	764	1080	1700
8	24 (12)	31 (6.5)	42 (3.2)	64 (1.3)	88 (.65)	123 (.33)	194 (.12)	271 (.06)	381 (.03)	599 (.01)	844	1190	1880
9	26 (13)	34 (7.3)	46 (3.6)	70 (1.4)	96 (.73)	134 (.37)	212 (.14)	297 (.07)	417 (.03)	655 (.01)	923	1300	2060
10	30 (13)	38 (7.3)	51 (3.6)	77 (1.4)	107 (.73)	148 (.37)	230 (.15)	322 (.08)	452 (.04)	711 (.01)	1000	1410	2230

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3a3

Sampling Plans for $\beta = 2/3$, $r = .50$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	100	50	25	10	5.0	2.5	1.0	.50	.25	.10	.05	.025	.010
0	4 (.25)	6 (.13)	9 (.07)	16 (.03)	25 (.02)	39 (.01)	72	114	182	334	529	838	1,550
1	7 (2.2)	10 (1.2)	15 (.68)	27 (.27)	42 (.13)	66 (.07)	122 (.03)	194 (.01)	307 (.01)	563	893	1,420	2,610
2	9 (5.6)	14 (2.7)	21 (1.5)	37 (.60)	58 (.30)	91 (.15)	168 (.06)	265 (.03)	419 (.01)	771	1,220	1,940	3,580
3	12 (8.2)	17 (4.5)	26 (2.3)	47 (.93)	73 (.47)	115 (.23)	211 (.09)	333 (.04)	526 (.02)	967 (.01)	1,530	2,430	4,490
4	14 (11)	21 (5.8)	31 (3.1)	56 (1.2)	87 (.62)	137 (.31)	253 (.12)	398 (.06)	630 (.03)	1,160 (.01)	1,840	2,910	5,370
5	17 (13)	24 (7.4)	37 (3.6)	65 (1.5)	102 (.75)	162 (.36)	293 (.15)	462 (.07)	731 (.03)	1,340 (.01)	2,130 (.01)	3,380	6,230
6	19 (16)	28 (8.4)	42 (4.1)	74 (1.7)	115 (.89)	184 (.43)	333 (.17)	524 (.09)	830 (.04)	1,520 (.02)	2,420 (.01)	3,830	7,070
7	21 (19)	31 (9.5)	47 (4.7)	82 (2.0)	129 (1.0)	205 (.48)	372 (.19)	586 (.10)	927 (.05)	1,710 (.02)	2,700 (.01)	4,280	7,900
8	24 (20)	34 (10)	52 (5.2)	91 (2.2)	143 (1.1)	226 (.54)	410 (.21)	647 (.10)	1,030 (.05)	1,880 (.02)	2,980 (.01)	4,730	8,720
9	26 (21)	38 (10)	57 (5.4)	100 (2.3)	159 (1.1)	248 (.58)	448 (.23)	707 (.11)	1,120 (.06)	2,060 (.02)	3,260 (.01)	5,170 (.01)	9,540
10	30 (22)	44 (10)	64 (5.5)	111 (2.3)	172 (1.2)	268 (.63)	487 (.25)	767 (.12)	1,220 (.06)	2,230 (.02)	3,540 (.01)	5,610 (.01)	

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3a4

Sampling Plans for $\beta = 1$, $r = .50$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	100	50	25	15	10	5.0	2.5	1.5	1.0	.50	.25	.15	.10
0	4 (1.8)	7 (1.0)	14 (.53)	23 (.33)	34 (.22)	67 (.11)	133 (.06)	224 (.03)	334 (.02)	664 (.01)	1,330	2,210	3,340
1	7 (7.7)	12 (4.4)	23 (2.3)	38 (1.3)	57 (.90)	113 (.45)	226 (.22)	378 (.13)	563 (.09)	1120 (.04)	2250 (.02)	3740 (.01)	5640 (.01)
2	9 (14)	17 (7.3)	32 (3.7)	53 (2.3)	78 (1.5)	156 (.75)	309 (.38)	517 (.23)	770 (.15)	1530 (.07)	3080 (.04)	5120 (.02)	7710 (.01)
3	12 (18)	21 (10)	40 (5.1)	66 (3.0)	98 (2.0)	196 (1.0)	388 (.51)	649 (.30)	967 (.20)	1930 (.10)	3860 (.05)	6420 (.03)	9680 (.02)
4	14 (23)	26 (12)	49 (6.0)	79 (3.7)	118 (2.4)	234 (1.2)	465 (.61)	776 (.36)	1160 (.24)	2300 (.12)	4620 (.06)	7690 (.04)	
5	17 (26)	30 (13)	56 (6.9)	92 (4.1)	137 (2.8)	272 (1.4)	539 (.70)	900 (.42)	1340 (.28)	2670 (.14)	5360 (.07)	8920 (.04)	
6	19 (30)	34 (15)	64 (7.6)	105 (4.6)	157 (3.0)	309 (1.5)	612 (.77)	1020 (.46)	1530 (.31)	3040 (.15)	6090 (.08)		
7	21 (33)	38 (16)	72 (8.3)	117 (5.0)	176 (3.2)	345 (1.6)	684 (.84)	1140 (.50)	1700 (.34)	3390 (.17)	6800 (.08)		
8	24 (34)	42 (18)	79 (8.9)	129 (5.3)	194 (3.5)	381 (1.8)	755 (.90)	1260 (.54)	1880 (.36)	3740 (.18)	7510 (.09)		
9	26 (35)	46 (18)	87 (9/3)	142 (5.6)	213 (3.7)	417 (1.9)	827 (.94)	1,380 (.57)	2,060 (.38)	4,100 (.19)	8,210 (.09)		
10	30 (36)	53 (18)	97 (9.7)	156 (5.7)	230 (3.9)	452 (1.9)	896 (.98)	1,500 (.59)	2,230 (.39)	4,440 (.20)	8,910 (.10)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3a5

Sampling Plans for $\beta = 1\ 1/3$, $r = .50$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	100	50	40	25	15	10	8.0	5.0	4.0	2.5	1.5	1.0	.50
0	4 (4.9)	9 (2.7)	12 (2.2)	22 (1.4)	42 (.86)	72 (.57)	97 (.46)	181 (.28)	242 (.23)	452 (.14)	900 (.08)	1540 (.06)	3840 (.03)
1	7 (15)	15 (8.0)	20 (6.4)	37 (4.0)	71 (2.5)	122 (1.6)	165 (1.3)	306 (.82)	409 (.66)	763 (.41)	1520 (.24)	2600 (.16)	6480 (.08)
2	9 (24)	21 (12)	27 (9.8)	50 (6.0)	98 (3.6)	168 (2.4)	226 (1.9)	419 (1.2)	560 (1.0)	1050 (.61)	2080 (.36)	3550 (.24)	8870 (.12)
3	12 (29)	26 (15)	35 (12)	63 (7.5)	123 (4.5)	211 (3.0)	283 (2.4)	526 (1.5)	703 (1.2)	1310 (.76)	2610 (.45)	4460 (.30)	
4	14 (34)	32 (17)	42 (13)	76 (8.6)	147 (5.2)	252 (3.4)	339 (2.7)	629 (1.7)	841 (1.4)	1570 (.88)	3120 (.52)	5330 (.35)	
5	17 (36)	37 (19)	48 (15)	88 (9.5)	173 (5.6)	293 (3.8)	393 (3.0)	730 (1.9)	976 (1.5)	1820 (.97)	3620 (.58)	6180 (.38)	
6	19 (41)	42 (20)	55 (16)	100 (10)	196 (6.1)	332 (4.1)	446 (3.3)	829 (2.1)	1110 (1.6)	2070 (1.0)	4110 (.62)	7020 (.41)	
7	21 (44)	47 (21)	61 (17)	112 (11)	219 (6.6)	371 (4.3)	499 (3.5)	927 (2.2)	1240 (1.7)	2310 (1.1)	4600 (.66)	7850 (.45)	
8	24 (45)	52 (23)	68 (18)	124 (11)	242 (6.8)	410 (4.6)	550 (3.6)	1020 (2.3)	1370 (1.8)	2550 (1.1)	5080 (.70)	8660 (.46)	
9	26 (46)	57 (24)	74 (19)	135 (12)	265 (7.1)	448 (4.8)	602 (3.8)	1120 (2.4)	1500 (1.9)	2790 (1.2)	5550 (.72)	9470 (.48)	
10	29 (46)	63 (24)	82 (19)	150 (12)	287 (7.4)	486 (5.0)	653 (4.0)	1210 (2.5)	1620 (2.0)	3020 (1.2)	6020 (.75)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3a6

Sampling Plans for $\beta = 1\ 2/3$, $r = .50$

c	n (t/ρ) x 100 Ratio for which P(A) = .10 or less												
	100	80	50	40	25	15	10	8.0	5.0	4.0	2.5	1.5	1.0
0	4 (9.0)	5 (7.9)	11 (4.9)	16 (3.9)	35 (2.4)	76 (1.5)	156 (1.0)	224 (.81)	480 (.51)	698 (.41)	1540 (.25)	3600 (.15)	7090 (.10)
1	7 (21)	9 (18)	19 (11)	27 (9.2)	59 (5.8)	129 (3.6)	263 (2.3)	378 (1.9)	810 (1.2)	1180 (.95)	2590 (.60)	6080 (.36)	
2	9 (31)	13 (24)	26 (16)	37 (12)	81 (7.8)	178 (4.8)	360 (3.2)	517 (2.6)	1110 (1.6)	1610 (1.3)	3550 (.81)	8320 (.49)	
3	12 (36)	16 (30)	33 (18)	46 (15)	102 (9.3)	223 (5.8)	451 (3.8)	650 (3.0)	1390 (1.9)	2030 (1.5)	4460 (.97)		
4	14 (42)	19 (34)	39 (21)	55 (17)	122 (10)	267 (6.4)	540 (4.2)	776 (3.4)	1670 (2.1)	2420 (1.7)	5330 (1.1)		
5	17 (44)	22 (37)	45 (23)	64 (18)	142 (11)	310 (7.0)	627 (4.6)	900 (3.7)	1930 (2.3)	2810 (1.8)	6180 (1.1)		
6	19 (48)	26 (38)	52 (24)	73 (19)	163 (12)	352 (7.4)	711 (4.9)	1020 (3.9)	2200 (2.5)	3190 (2.0)	7020 (1.2)		
7	21 (51)	29 (40)	58 (25)	82 (20)	182 (12)	394 (7.8)	795 (5.1)	1140 (4.1)	2450 (2.6)	3570 (2.1)	7850 (1.3)		
8	24 (52)	32 (42)	65 (26)	90 (21)	201 (13)	434 (8.2)	878 (5.3)	1260 (4.3)	2710 (2.7)	3940 (2.1)	8660 (1.3)		
9	26 (53)	35 (44)	70 (27)	99 (22)	220 (13)	475 (8.4)	960 (5.5)	1380 (4.4)	2960 (2.8)	4310 (2.2)	9470 (1.4)		
10	29 (54)	39 (44)	78 (27)	110 (22)	239 (14)	515 (8.7)	1040 (5.7)	1500 (4.6)	3210 (2.9)	4670 (2.3)			

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3a7

Sampling Plans for β = 2, r = .50

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	100	80	50	40	25	15	10	8.0	5.0	4.0	3.0	2.5	2.0
0	4 (13)	6 (11)	14 (7.2)	21 (5.9)	54 (3.6)	147 (2.2)	334 (1.5)	520 (1.2)	1330 (.74)	2080 (.60)	3710 (.45)	5360 (.37)	8220 (.28)
1	7 (28)	10 (23)	23 (15)	36 (12)	91 (7.5)	251 (4.5)	563 (3.0)	878 (2.4)	2250 (1.5)	3500 (1.2)	6280 (.90)	9050 (.75)	
2	9 (38)	14 (30)	32 (19)	49 (15)	124 (9.8)	343 (5.8)	770 (4.1)	1200 (3.1)	3080 (1.9)	4800 (1.5)	8590 (1.1)		
3	12 (43)	17 (35)	40 (22)	62 (18)	158 (11)	431 (6.7)	967 (4.5)	1510 (3.6)	3860 (2.2)	6020 (1.8)			
4	14 (48)	21 (38)	49 (24)	74 (20)	189 (13)	516 (7.4)	1160 (5.0)	1810 (3.9)	4620 (2.4)	7200 (1.9)			
5	17 (50)	24 (41)	56 (26)	86 (21)	219 (13)	598 (7.9)	1340 (5.3)	2090 (4.2)	5360 (2.6)	8360 (2.1)			
6	19 (54)	27 (44)	64 (27)	98 (22)	248 (14)	679 (8.4)	1520 (5.5)	2380 (4.4)	6090 (2.7)	9490 (2.2)			
7	21 (57)	31 (45)	72 (28)	110 (23)	278 (14)	759 (8.7)	1700 (5.8)	2660 (4.6)	6800 (2.8)				
8	24 (58)	34 (47)	79 (30)	121 (24)	306 (15)	838 (9.0)	1880 (6.0)	2930 (4.8)	7510 (2.9)				
9	26 (58)	37 (47)	87 (31)	132 (25)	335 (15)	917 (9.2)	2,060 (6.1)	3,210 (4.9)	8,210 (3.0)				
10	30 (58)	43 (47)	97 (31)	147 (25)	363 (16)	994 (9.4)	2230 (6.3)	3480 (5.0)	8910 (3.1)				

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3a8

Sampling Plans for $\beta = 2\ 1/2$, $r = .50$

c	n												
	$(t/\rho) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	100	80	65	50	40	30	25	20	15	12	10	8.0	5.0
0	4 (20)	6 (17)	10 (14)	19 (10)	33 (8.6)	68 (6.5)	107 (6.5)	187 (4.3)	378 (3.3)	662 (2.6)	1000 (2.2)	1840 (1.7)	5910 (1.1)
1	7 (36)	11 (29)	17 (24)	33 (18)	56 (15)	115 (11)	182 (9.4)	316 (7.6)	638 (5.7)	1120 (4.6)	1690 (3.8)	3110 (3.0)	9980 (1.9)
2	9 (46)	15 (27)	24 (30)	45 (23)	77 (18)	158 (14)	249 (11)	433 (9.3)	872 (7.1)	1530 (5.6)	2320 (4.8)	4260 (3.7)	
3	12 (51)	19 (41)	30 (34)	56 (26)	97 (20)	199 (15)	312 (13)	543 (10)	1100 (7.9)	1920 (6.3)	2910 (5.3)	5350 (4.2)	
4	14 (56)	23 (44)	36 (37)	68 (28)	116 (22)	238 (17)	374 (14)	650 (11)	1310 (8.5)	2300 (6.8)	3480 (5.8)	6400 (4.5)	
5	17 (58)	26 (48)	42 (38)	79 (29)	135 (23)	276 (18)	433 (15)	754 (12)	1520 (9.0)	2670 (7.2)	4030 (6.1)	7420 (4.8)	
6	19 (61)	30 (50)	48 (40)	89 (31)	155 (24)	313 (18)	492 (15)	856 (12)	1730 (9.3)	3030 (7.5)	4580 (6.3)	8430 (5.0)	
7	21 (63)	34 (51)	54 (41)	100 (32)	174 (25)	350 (19)	550 (16)	957 (12)	1930 (9.7)	3380 (7.7)	5120 (6.6)	9420 (5.1)	
8	24 (64)	37 (52)	60 (43)	110 (32)	192 (26)	387 (19)	607 (16)	1060 (13)	2130 (10)	3730 (7.9)	5650 (6.7)		
9	26 (65)	41 (53)	65 (43)	121 (33)	210 (26)	423 (20)	664 (16)	1160 (13)	2330 (10)	4080 (8.1)	6180 (6.9)		
10	29 (65)	46 (53)	72 (43)	134 (34)	227 (27)	459 (20)	720 (17)	1250 (13)	2530 (10)	4430 (8.2)	6670 (7.0)		

$(t/\rho) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

Table 3b1

Sampling Plans for $\beta = 1/3$, $r = .90$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	100	50	25	10	5.0	2.5	1.0	.50	.25	.10	.050	.025	.010
0	22	28	35	48	60	76	104	129	163	220	277	348	470
1	38 (.08)	48 (.03)	60 (.02)	82 (.01)	101	129	174	217	274	370	467	587	794
2	52 (.35)	65 (.18)	82 (.08)	112 (.03)	139 (.02)	176 (.01)	237	297	375	507	639	804	1,090
3	65 (.84)	82 (.41)	103 (.21)	141 (.08)	175 (.04)	220 (.02)	298 (.01)	373	470	636	802	1,010	1,360
4	78 (1.4)	98 (.73)	123 (.37)	169 (.14)	210 (.07)	264 (.03)	357 (.01)	446 (.01)	563	761	960	1,210	1,630
5	91 (2.2)	114 (1.1)	143 (.54)	196 (.20)	243 (.10)	306 (.05)	414 (.02)	518 (.01)	653	883	1,110	1,400	1,890
6	103 (2.9)	130 (1.4)	164 (.71)	223 (.27)	276 (.14)	347 (.07)	470 (.03)	588 (.01)	741 (.01)	1,000	1,260	1,590	2,150
7	115 (3.8)	145 (1.9)	184 (.89)	250 (.35)	309 (.18)	389 (.09)	526 (.03)	658 (.02)	829 (.01)	1120	1410	1780	2400
8	127 (4.7)	162 (2.1)	202 (1.0)	275 (.43)	341 (.22)	429 (.11)	580 (.04)	726 (.02)	915 (.01)	1,240	1,560	1,960	2,650
9	139 (5.5)	177 (2.5)	221 (1.2)	301 (.51)	373 (.26)	469 (.13)	634 (.05)	794 (.03)	1,000 (.01)	1,350	1,710	2,150	2,900
10	150 (5.8)	193 (2.9)	240 (1.4)	327 (.58)	405 (.29)	509 (.15)	688 (.06)	861 (.03)	1090 (.01)	1470	1850	2330	3150

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b2

Sampling Plans for $\beta = 1/2$, $r = .90$

c	n												
	$(t/\rho) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	100	50	25	10	5.0	2.5	1.0	.50	.25	.10	.050	.025	.010
0	22 (.05)	31 (.02)	45 (.01)	69	98	138	219	307	435	687	980	1400	2190
1	38 (.80)	53 (.40)	75 (.20)	117 (.08)	167 (.04)	236 (.02)	370 (.01)	519	734	1160	1660	2360	3710
2	52 (2.3)	73 (1.2)	102 (.59)	162 (.23)	228 (.11)	323 (.06)	507 (.02)	710 (.01)	1000	1590	2270	3230	5070
3	65 (3.9)	92 (2.0)	129 (1.0)	204 (.40)	287 (.20)	405 (.10)	636 (.04)	891 (.02)	1260 (.01)	1990	2840	4050	6360
4	78 (6.0)	110 (3.0)	156 (1.4)	244 (.59)	343 (.30)	484 (.15)	761 (.06)	1070 (.03)	1510 (.01)	2390	3400	4850	7610
5	91 (8.0)	127 (4.0)	181 (1.9)	283 (.76)	398 (.38)	562 (.19)	883 (.08)	1240 (.04)	1750 (.02)	2770 (.01)	3950	5620	8830
6	103 (9.6)	145 (4.8)	205 (2.3)	321 (.94)	452 (.47)	638 (.24)	1000 (.09)	1400 (.04)	1990 (.02)	3140 (.01)	4480	6380	
7	115 (11)	164 (5.4)	229 (2.8)	359 (1.1)	505 (.55)	713 (.28)	1120 (.11)	1570 (.06)	2220 (.03)	3510 (.01)	5010	7130	
8	127 (13)	181 (6.2)	253 (3.1)	396 (1.3)	558 (.63)	787 (.32)	1240 (.13)	1730 (.06)	2450 (.03)	3880 (.01)	5530	7870	
9	139 (15)	198 (6.9)	277 (3.4)	433 (1.4)	610 (.71)	861 (.35)	1350 (.14)	1900 (.07)	2680 (.04)	4240 (.01)	6050 (.01)	8610	
10	154 (15)	215 (7.6)	300 (3.8)	470 (1.5)	661 (.77)	934 (.38)	1470 (.16)	2060 (.08)	2910 (.04)	4600 (.01)	6560 (.01)	9340	

$(t/\rho) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

Table 3b3

Sampling Plans for β = 2/3 r = .90

c	n (t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	200	100	50	25	15	10	5.0	2.5	1.0	.50	.25	.10	.05
0	14 (.63)	22 (.33)	35 (.16)	55 (.08)	78 (.05)	102 (.03)	162 (.02)	259 (.01)	470	743	1180	2170	3440
1	24 (5.4)	38 (2.7)	59 (1.4)	94 (.68)	131 (.41)	173 (.27)	274 (.13)	437 (.07)	794 (.03)	1260 (.01)	2000	3670	5810
2	33 (11)	52 (5.9)	82 (2.9)	128 (1.5)	181 (.87)	237 (.59)	375 (.29)	598 (.14)	1090 (.06)	1720 (.03)	2730 (.01)	5020	7940
3	42 (18)	65 (8.8)	103 (4.6)	163 (2.2)	227 (1.4)	297 (.90)	471 (.46)	751 (.22)	1360 (.09)	2160 (.04)	3430 (.02)	6300 (.01)	9970
4	50 (24)	78 (12)	123 (6.1)	195 (3.0)	272 (1.8)	355 (1.2)	563 (.60)	898 (.30)	1630 (.12)	2580 (.06)	4100 (.03)	7540 (.01)	
5	58 (30)	91 (15)	143 (7.4)	226 (3.6)	315 (2.2)	412 (1.5)	653 (.74)	1040 (.36)	1890 (.14)	2990 (.07)	4760 (.04)	8750 (.01)	
6	66 (35)	103 (17)	164 (8.4)	257 (4.3)	358 (2.6)	468 (1.7)	742 (.86)	1180 (.42)	2150 (.18)	3400 (.09)	5400 (.04)	9930 (.02)	
7	74 (39)	115 (20)	183 (9.5)	287 (4.8)	400 (2.9)	523 (1.9)	829 (.96)	1320 (.48)	2400 (.20)	3800 (.10)	6040 (.05)		
8	82 (43)	127 (22)	202 (10)	317 (5.3)	442 (3.2)	578 (2.1)	915 (1.0)	1460 (.52)	2650 (.21)	4190 (.10)	6660 (.05)		
9	90 (47)	139 (23)	221 (11)	347 (5.7)	483 (3.5)	632 (2.3)	1000 (1.1)	1600 (.58)	2900 (.24)	4580 (.12)	7290 (.06)		
10	100 (47)	154 (24)	240 (12)	376 (6.2)	524 (3.8)	685 (2.5)	1090 (1.2)	1730 (.62)	3150 (.25)	4970 (.12)	7900 (.06)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b4

Sampling Plans for $\beta = 1$, $r = .90$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	200	150	100	80	50	25	15	10	8.0	5.0	2.5	1.5	1.0
0	11 (4.4)	15 (3.3)	22 (2.2)	28 (1.7)	44 (1.1)	88 (.55)	146 (.33)	219 (.22)	274 (.18)	438 (.11)	876 (.05)	1,460 (.03)	2,190 (.02)
1	19 (18)	26 (13)	38 (9.0)	47 (7.2)	75 (4.5)	148 (2.2)	248 (1.3)	370 (.92)	463 (.73)	740 (.45)	1,480 (.22)	2,460 (.13)	3,710 (.09)
2	27 (29)	35 (22)	52 (15)	65 (12)	102 (7.7)	205 (3.8)	339 (2.2)	507 (1.5)	634 (1.2)	1,010 (.77)	2,020 (.38)	3,370 (.22)	5,070 (.15)
3	34 (39)	44 (30)	65 (20)	81 (16)	129 (10)	257 (5.0)	426 (3.0)	636 (2.0)	795 (1.6)	1,270 (1.0)	2,540 (.50)	4,230 (.30)	6,360 (.20)
4	40 (49)	53 (36)	78 (24)	97 (19)	156 (12)	307 (6.0)	509 (3.6)	761 (2.4)	952 (1.9)	1,520 (1.2)	3,040 (.61)	5,060 (.36)	7,610 (.24)
5	47 (55)	62 (41)	91 (29)	113 (22)	181 (14)	357 (7.0)	591 (4.2)	883 (2.8)	1,100 (2.2)	1,760 (1.4)	3,530 (.70)	5,870 (.41)	8,830 (.28)
6	53 (61)	70 (46)	103 (30)	128 (25)	205 (15)	405 (7.7)	671 (4.6)	1,000 (3.1)	1,250 (2.5)	2,000 (1.5)	4,000 (.78)	6,670 (.46)	
7	60 (66)	78 (51)	115 (33)	143 (27)	229 (16)	453 (8.4)	750 (5.0)	1,120 (3.3)	1,400 (2.7)	2,240 (1.6)	4,480 (.85)	7,450 (.50)	
8	67 (70)	87 (53)	127 (36)	161 (28)	253 (18)	500 (9.0)	827 (5.4)	1,240 (3.6)	1,550 (2.9)	2,470 (1.8)	4,940 (.90)	8,220 (.54)	
9	72 (75)	95 (57)	139 (38)	176 (30)	277 (19)	547 (9.5)	905 (5.7)	1,350 (3.8)	1,690 (3.0)	2,700 (1.9)	5,400 (.95)	8,990 (.57)	
10	80 (76)	106 (58)	155 (39)	191 (31)	301 (20)	593 (10)	982 (6.0)	1,470 (4.0)	1,840 (3.1)	2,930 (2.0)	5,860 (1.0)	9,760 (.6)	

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b5

Sampling Plans for $\beta = 1\ 1/3$, $r = .90$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	200	150	100	80	50	40	25	15	10	8.0	5.0	4.0	2.5
0	9 (11)	13 (8.3)	22 (5.6)	30 (4.5)	55 (2.8)	74 (2.3)	138 (1.4)	274 (.87)	461 (.58)	622 (.45)	1150 (.29)	1540 (.23)	2950 (.14)
1	16 (31)	22 (25)	38 (16)	51 (13)	94 (8.0)	126 (6.5)	236 (4.1)	463 (2.5)	778 (1.7)	1050 (1.3)	1950 (.83)	2590 (.69)	4990 (.45)
2	21 (49)	31 (36)	52 (24)	69 (19)	128 (12)	174 (9.5)	323 (6.0)	634 (3.6)	1070 (2.5)	1440 (2.0)	2660 (1.2)	3550 (1.0)	6820 (.62)
3	27 (59)	39 (45)	65 (30)	87 (24)	163 (15)	218 (12)	405 (7.4)	795 (4.5)	1340 (3.0)	1810 (2.4)	3340 (1.5)	4450 (1.2)	8570 (.76)
4	33 (67)	46 (52)	78 (34)	105 (27)	195 (17)	261 (13)	485 (8.5)	952 (5.1)	1600 (3.5)	2160 (2.8)	4000 (1.7)	5330 (1.4)	
5	38 (75)	54 (57)	91 (38)	121 (29)	226 (19)	303 (15)	562 (9.5)	1100 (5.7)	1860 (3.9)	2510 (3.1)	4640 (1.9)	6180 (1.6)	
6	43 (81)	61 (61)	103 (41)	138 (33)	257 (20)	344 (16)	638 (10)	1250 (6.2)	2110 (4.2)	2850 (3.3)	5270 (2.1)	7020 (1.7)	
7	48 (87)	69 (66)	115 (44)	156 (34)	287 (22)	385 (17)	713 (11)	1400 (6.5)	2360 (4.4)	3180 (3.5)	5890 (2.2)	7850 (1.8)	
8	54 (90)	76 (69)	127 (46)	173 (36)	317 (23)	425 (18)	787 (11)	1550 (6.9)	2600 (4.7)	3510 (3.7)	6500 (2.3)	8660 (1.9)	
9	59 (94)	83 (72)	139 (49)	189 (37)	347 (24)	464 (19)	861 (12)	1690 (7.2)	2840 (4.9)	3840 (3.9)	7110 (2.4)	9470 (1.9)	
10	66 (95)	93 (72)	154 (49)	205 (39)	376 (25)	504 (19)	934 (12)	1840 (7.4)	3080 (5.0)	4170 (4.0)	7710 (2.5)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b6

Sampling Plans for $\beta = 1\ 2/3,\ r = .90$

c	n: (t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	200	150	100	80	65	50	40	25	15	10	8.0	5.0	4.0
0	7 (20)	12 (14)	22 (10)	32 (8.1)	45 (6.6)	70 (5.0)	101 (4.1)	221 (2.5)	518 (1.5)	1010 (1.0)	1470 (.85)	3200 (.50)	4650 (.40)
1	13 (45)	20 (35)	38 (23)	54 (19)	77 (15)	118 (12)	172 (9.4)	374 (5.9)	874 (3.6)	1710 (2.4)	2480 (1.8)	5400 (1.2)	7860 (.95)
2	17 (65)	27 (48)	52 (32)	75 (26)	105 (21)	163 (16)	235 (13)	512 (8.1)	1200 (4.8)	2330 (3.2)	3390 (2.6)	7390 (1.6)	
3	22 (75)	34 (57)	65 (38)	94 (30)	132 (25)	205 (19)	296 (15)	642 (9.6)	1500 (5.7)	2930 (3.9)	4260 (3.1)	9280 (1.9)	
4	26 (85)	41 (64)	78 (43)	112 (34)	160 (28)	245 (21)	354 (17)	769 (11)	1800 (6.5)	3510 (4.3)	5090 (3.4)		
5	31 (90)	48 (69)	91 (46)	131 (37)	185 (30)	285 (23)	410 (18)	892 (11)	2090 (7.0)	4070 (4.6)	5910 (3.7)		
6	35 (97)	54 (74)	103 (48)	148 (39)	210 (32)	323 (24)	466 (19)	1010 (12)	2370 (7.4)	4620 (4.9)	6710 (3.9)		
7	39 (105)	61 (77)	115 (52)	168 (41)	235 (33)	361 (26)	521 (20)	1130 (13)	2650 (7.8)	5160 (5.2)	7500 (4.1)		
8	44 (108)	68 (80)	127 (54)	185 (43)	259 (35)	398 (27)	575 (21)	1250 (13)	2920 (8.1)	5700 (5.4)	8280 (4.3)		
9	48 (110)	74 (83)	139 (56)	203 (44)	284 (36)	439 (27)	629 (22)	1370 (14)	3190 (8.4)	6230 (5.6)	9050 (4.5)		
10	53 (110)	82 (83)	154 (57)	220 (45)	308 (37)	473 (28)	682 (23)	1480 (14)	3460 (8.7)	6760 (5.7)	9820 (4.6)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b7

Sampling Plans for $\beta = 2$, $r = .90$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 (or less)												
	200	150	120	100	80	65	50	40	25	20	15	10	8.0
0	6 (28)	10 (22)	15 (18)	22 (14)	34 (12)	52 (9.6)	88 (7.4)	137 (5.9)	349 (3.7)	548 (2.9)	960 (2.2)	2170 (1.5)	3390 (1.2)
1	10 (60)	17 (45)	26 (36)	38 (30)	59 (24)	88 (19)	148 (15)	233 (12)	589 (7.5)	926 (5.9)	1620 (4.5)	3670 (3.0)	5720 (2.4)
2	14 (77)	24 (58)	36 (47)	52 (39)	80 (31)	121 (25)	205 (19)	319 (15)	806 (9.7)	1270 (7.8)	2220 (5.9)	5020 (3.8)	7830 (3.0)
3	18 (88)	30 (68)	46 (54)	65 (45)	101 (36)	154 (29)	257 (22)	400 (18)	1010 (11)	1590 (9.0)	2780 (6.7)	6300 (4.5)	9830 (3.6)
4	21 (99)	36 (74)	55 (59)	78 (50)	121 (39)	184 (31)	307 (24)	479 (20)	1210 (12)	1900 (9.9)	3330 (7.4)	7540 (4.9)	
5	25 (105)	42 (79)	64 (63)	91 (53)	140 (41)	213 (34)	357 (26)	555 (21)	1410 (13)	2210 (10)	3870 (7.9)	8750 (5.3)	
6	29 (110)	48 (83)	73 (66)	103 (56)	161 (44)	242 (36)	405 (28)	631 (22)	1600 (14)	2510 (11)	4390 (8.4)	9930 (5.5)	
7	32 (115)	54 (86)	81 (70)	115 (58)	181 (46)	271 (37)	453 (29)	705 (23)	1780 (14)	2800 (11)	4900 (8.7)		
8	36 (118)	60 (89)	90 (72)	127 (60)	199 (47)	299 (38)	500 (30)	778 (24)	1970 (15)	3090 (12)	5410 (9.0)		
9	39 (123)	65 (91)	98 (74)	139 (62)	218 (49)	327 (40)	546 (31)	851 (24)	2150 (15)	3380 (12)	5920 (9.3)		
10	45 (123)	73 (91)	109 (74)	154 (62)	236 (50)	354 (41)	593 (31)	923 (25)	2340 (16)	3670 (12)	6240 (9.6)		

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3b8

Sampling Plans for $\beta = 2\ 1/2$, $r = .90$

c	n $(t/\rho) \times 100$ Ratio for which $P(A) = .10$ (or less) 200	150	120	100	80	65	50	40	30	25	20	15	12
0	4 (43)	8 (32)	14 (26)	22 (21)	39 (17)	64 (14)	124 (11)	217 (8.7)	439 (6.5)	700 (5.3)	1210 (4.3)	2480 (3.2)	4300 (2.6)
1	8 (72)	14 (57)	24 (46)	38 (38)	65 (30)	109 (24)	211 (19)	367 (15)	741 (11)	1180 (9.5)	2050 (7.6)	4180 (5.7)	7270 (4.5)
2	10 (94)	20 (70)	33 (57)	52 (47)	90 (37)	149 (30)	289 (23)	502 (19)	1010 (14)	1610 (12)	2800 (9.4)	5720 (7.0)	9950 (5.6)
3	13 (105)	25 (79)	42 (63)	65 (53)	113 (42)	189 (34)	363 (26)	630 (21)	1270 (14)	2030 (13)	3520 (10)	7180 (7.9)	
4	16 (113)	30 (85)	51 (68)	78 (57)	135 (45)	226 (37)	434 (28)	754 (23)	1520 (17)	2420 (14)	4210 (11)	8600 (8.6)	
5	19 (118)	35 (89)	59 (72)	91 (60)	159 (47)	263 (38)	504 (30)	875 (24)	1770 (18)	2810 (15)	4880 (12)	9970 (9.0)	
6	21 (125)	40 (93)	67 (75)	103 (63)	180 (50)	298 (40)	572 (31)	993 (25)	2010 (19)	3190 (15)	5540 (12)		
7	24 (128)	45 (96)	75 (77)	115 (65)	201 (51)	333 (42)	640 (32)	1110 (25)	2240 (19)	3570 (16)	6200 (13)		
8	27 (131)	49 (99)	83 (79)	127 (67)	222 (53)	368 (43)	706 (33)	1230 (26)	2470 (20)	3940 (16)	6840 (13)		
9	29 (135)	54 (100)	90 (81)	139 (68)	243 (54)	403 (44)	772 (34)	1340 (27)	2710 (20)	4310 (17)	7480 (13)		
10	33 (129)	60 (101)	100 (82)	154 (69)	263 (55)	437 (45)	838 (34)	1450 (27)	2940 (21)	4670 (17)	8110 (14)		

$(t/\rho) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

Table 3c1

Sampling Plans for $\beta = 1/3$, $r = .99$

c	n: $(t/\rho) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	1500	1000	500	200	100	50	10	5.0	1.0	.50	.10	.050	.010
0	93 (.01)	107 (.01)	134	183	230	288	495	622	1070	1360	2300	2880	4950
1	159 (1.1)	182 (.75)	229 (.37)	309 (.15)	389 (.07)	486 (.04)	837 (.01)	1050	1800	2290	3890	4860	8370
2	217 (5.4)	249 (3.5)	313 (1.8)	422 (.73)	532 (.37)	665 (.18)	1150 (.04)	1440 (.02)	2470	3130	5320	6650	
3	273 (12)	312 (8.4)	393 (4.2)	530 (1.7)	668 (.87)	835 (.44)	1440 (.08)	1810 (.04)	3090 (.01)	3930	6680	8350	
4	326 (22)	374 (15)	470 (7.4)	634 (3.0)	799 (1.5)	999 (.76)	1720 (.15)	2160 (.07)	3700 (.01)	4700	8000	9990	
5	379 (32)	433 (22)	546 (11)	736 (4.5)	928 (2.2)	1160 (1.1)	2000 (.22)	2510 (.11)	4300 (.02)	5460 (.01)	9280		
6	430 (44)	492 (30)	619 (15)	836 (6.1)	1050 (2.8)	1320 (1.5)	2270 (.30)	2850 (.15)	4880 (.03)	6200 (.01)			
7	480 (56)	550 (37)	692 (18)	934 (7.8)	1180 (3.8)	1470 (2.0)	2530 (.38)	3180 (.19)	5450 (.04)	6930 (.02)			
8	530 (68)	607 (46)	764 (23)	1030 (9.4)	1300 (4.7)	1630 (2.4)	2790 (.48)	3510 (.24)	6020 (.05)	7640 (.02)			
9	580 (80)	664 (53)	836 (27)	1130 (11)	1420 (5.6)	1780 (2.8)	3060 (.55)	3840 (.28)	6580 (.06)	8360 (.03)			
10	629 (93)	720 (62)	906 (31)	1220 (13)	1540 (6.4)	1930 (3.2)	3320 (.65)	4170 (.32)	7140 (.06)	9070 (.03)			

$(t/\rho) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

Table 3c2

Sampling Plans for $\beta = 1/2$, r = .99

c	n (t/) x 100 Ratio for which P(A) = .10 (or less) 1500	1000	500	200	100	50	25	10	5.0	2.5	1.0	.50	.10
0	60 (.71)	73 (.50)	103 (.24)	163 (.10)	230 (.05)	324 (.02)	461 (.01)	683	1030	1540	2330	3250	6840
1	101 (12)	123 (8.1)	175 (4.0)	276 (1.6)	389 (.82)	548 (.41)	778 (.20)	1220 (.08)	1730 (.04)	2430 (.02)	3930 (.01)	5480	
2	138 (35)	170 (23)	240 (11)	377 (4.6)	532 (2.3)	750 (1.2)	1070 (.59)	1660 (.24)	2370 (.12)	3330 (.06)	5380 (.02)	7500 (.01)	
3	175 (60)	213 (40)	301 (20)	474 (8.1)	668 (4.1)	941 (2.0)	1340 (1.0)	2090 (.42)	2970 (.21)	4180 (.10)	6750 (.04)	9410 (.02)	
4	209 (88)	255 (59)	360 (30)	567 (12)	799 (5.9)	1130 (3.0)	1600 (1.5)	2500 (.62)	3550 (.30)	5000 (.15)	8080 (.06)		
5	243 (125)	296 (77)	418 (38)	658 (15)	928 (7.8)	1310 (4.0)	1860 (1.9)	2900 (.80)	4120 (.38)	5800 (.20)	9370 (.08)		
6	276 (140)	336 (94)	474 (47)	747 (19)	1050 (9.6)	1480 (4.8)	2110 (2.4)	3290 (1.0)	4680 (.50)	6580 (.25)			
7	308 (165)	376 (106)	530 (55)	835 (22)	1180 (11)	1660 (5.6)	2360 (2.8)	3680 (1.1)	5230 (.57)	7360 (.28)			
8	340 (190)	415 (125)	585 (65)	921 (26)	1300 (13)	1830 (6.5)	2600 (3.2)	4060 (1.3)	5770 (.65)	8120 (.33)			
9	372 (210)	454 (140)	640 (71)	1010 (28)	1420 (14)	2000 (7.2)	2840 (3.6)	4440 (1.4)	6320 (.73)	8880 (.37)			
10	403 (230)	492 (155)	694 (80)	1090 (31)	1540 (16)	2170 (8.0)	3080 (4.0)	4820 (1.6)	6850 (.80)	9630 (.40)			

$(t/\rho) \times 100$ ratios in parentheses are for P(A) = .95 (or more)

Table 3c3

Sampling Plans for $\beta = 2/3$, $r = .99$

c	n												
	(t/ρ) x 100 Ratio for which P(A) = .10 or less												
	1500	1000	500	300	200	100	50	25	15	10	5.0	2.5	1.0
0	38 (4.9)	49 (3.3)	79 (1.6)	109 (1.0)	143 (.67)	230 (.33)	366 (.16)	576 (.08)	808 (.05)	1060 (.03)	1690 (.01)	2710	4900
1	64 (41)	84 (30)	133 (14)	185 (8.3)	243 (5.4)	389 (2.8)	617 (1.4)	973 (.69)	1370 (.42)	1790 (.27)	2860 (.14)	4580 (.07)	8280 (.02)
2	87 (91)	114 (60)	181 (30)	253 (18)	333 (12)	532 (6.0)	845 (3.0)	1330 (1.5)	1870 (.90)	2450 (.60)	3910 (.29)	6260 (.15)	
3	110 (137)	144 (92)	227 (46)	318 (27)	418 (18)	668 (9.1)	1060 (4.5)	1670 (2.3)	2340 (1.4)	3080 (.91)	4910 (.45)	7860 (.22)	
4	132 (184)	174 (115)	271 (61)	381 (36)	500 (24)	800 (12)	1270 (6.0)	2000 (3.0)	2800 (1.8)	3680 (1.2)	5880 (.60)	9400 (.30)	
5	155 (215)	202 (145)	314 (75)	442 (45)	580 (30)	928 (15)	1470 (7.4)	2320 (3.7)	3250 (2.3)	4270 (1.5)	6820 (.74)		
6	176 (250)	229 (170)	357 (87)	501 (52)	658 (35)	1050 (17)	1670 (8.6)	2630 (4.4)	3700 (2.6)	4850 (1.7)	7740 (.87)		
7	196 (290)	256 (190)	399 (99)	561 (59)	736 (39)	1180 (19)	1870 (9.7)	2940 (4.9)	4130 (2.9)	5420 (1.9)	8650 (.97)		
8	217 (315)	282 (212)	440 (105)	619 (65)	812 (43)	1300 (21)	2060 (10)	3250 (5.4)	4560 (3.3)	5990 (2.1)	9550 (1.0)		
9	237 (345)	309 (230)	482 (117)	677 (71)	888 (47)	1420 (23)	2260 (10)	3550 (5.8)	4990 (3.4)	6550 (2.3)			
10	257 (370)	335 (250)	522 (126)	734 (76)	963 (50)	1540 (25)	2450 (12)	3850 (6.3)	5410 (3.8)	7100 (2.5)			

(t/ρ) x 100 ratios in parentheses are for P(A) = .95 (or more)

Table 3c4

Sampling Plans for $\beta = 1$, $r = .99$

c	n												
	$(t/\rho) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	1500	1000	800	500	300	200	100	80	50	25	15	10	5.0
0	16 (31)	23 (22)	29 (17)	46 (11)	77 (6.6)	114 (4.5)	230 (2.1)	291 (1.7)	461 (1.1)	903 (.57)	1520 (.34)	2260 (.23)	4610 (.11)
1	27 (132)	40 (88)	49 (72)	78 (45)	130 (27)	194 (18)	389 (9.1)	492 (7.2)	778 (4.6)	1530 (2.3)	2560 (1.4)	3820 (.93)	7780 (.46)
2	37 (225)	54 (153)	68 (120)	107 (76)	179 (45)	266 (30)	532 (15)	674 (12)	1070 (7.7)	2090 (3.9)	3500 (2.3)	5220 (1.6)	
3	46 (305)	68 (203)	85 (162)	135 (100)	225 (60)	334 (40)	668 (20)	846 (16)	1340 (10)	2680 (5.2)	4400 (3.1)	6550 (2.1)	
4	55 (370)	82 (245)	102 (195)	163 (120)	269 (73)	400 (49)	799 (24)	1010 (19)	1600 (12)	3140 (6.3)	5260 (3.7)	7840 (2.5)	
5	64 (422)	95 (280)	118 (225)	189 (138)	312 (83)	464 (56)	928 (28)	1180 (22)	1860 (14)	3640 (7.2)	6100 (4.3)	9100 (2.9)	
6	73 (470)	108 (310)	134 (250)	215 (153)	355 (92)	526 (62)	1060 (31)	1340 (24)	2110 (15)	4130 (7.9)	6930 (4.7)		
7	82 (510)	121 (335)	150 (270)	240 (166)	396 (100)	588 (67)	1180 (33)	1490 (26)	2360 (16)	4620 (8.6)	7750 (5.1)		
8	90 (550)	134 (360)	169 (280)	265 (175)	437 (105)	649 (72)	1300 (36)	1650 (28)	2600 (18)	5100 (9.2)	8550 (5.5)		
9	99 (570)	146 (380)	184 (298)	290 (189)	478 (112)	710 (76)	1420 (38)	1800 (30)	2840 (19)	5570 (9.7)	9350 (5.8)		
10	110 (580)	161 (390)	200 (310)	314 (197)	519 (118)	770 (79)	1540 (39)	1950 (31)	3080 (20)	6050 (10)			

$(t/\rho) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

Table 3c5

Sampling Plans for $\beta = 1\ 1/3$, $r = .99$

c	n												
	$(t/\rho) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	1500	1000	800	500	300	200	150	100	80	50	25	15	10
0	7 (77)	11 (56)	15 (44)	27 (28)	53 (17)	91 (11)	134 (8.5)	230 (5.7)	307 (4.6)	576 (2.8)	1460 (1.4)	2880 (.87)	4900 (.58)
1	11 (245)	19 (162)	26 (127)	46 (81)	90 (49)	155 (32)	227 (24)	389 (16)	519 (13)	973 (8.3)	2460 (4.1)	4860 (2.5)	8280 (1.6)
2	16 (350)	26 (240)	34 (195)	63 (123)	124 (73)	212 (48)	311 (36)	532 (24)	709 (19)	1330 (12)	3370 (6.0)	6650 (3.6)	
3	20 (420)	33 (295)	43 (240)	80 (151)	157 (89)	267 (60)	391 (45)	668 (30)	891 (24)	1670 (15)	4230 (7.3)	8350 (4.4)	
4	24 (510)	39 (345)	52 (275)	95 (175)	187 (104)	320 (68)	467 (52)	799 (34)	1070 (28)	2000 (17)	5060 (8.7)	9990 (5.3)	
5	28 (565)	46 (385)	60 (305)	110 (194)	217 (115)	371 (76)	542 (57)	928 (38)	1240 (31)	2320 (19)	5870 (9.7)		
6	32 (610)	52 (410)	69 (325)	126 (210)	247 (124)	421 (82)	616 (61)	1050 (41)	1410 (33)	2630 (21)	6670 (10)		
7	36 (650)	58 (440)	77 (350)	141 (220)	276 (132)	471 (87)	688 (65)	1180 (43)	1570 (35)	2940 (22)	7450 (11)		
8	40 (680)	65 (458)	85 (367)	157 (225)	305 (139)	520 (92)	760 (69)	1300 (46)	1730 (37)	3250 (23)	8220 (11)		
9	43 (720)	70 (485)	93 (383)	172 (235)	333 (144)	568 (95)	831 (72)	1420 (48)	1900 (38)	3550 (24)	9000 (12)		
10	47 (730)	78 (490)	103 (385)	187 (245)	362 (150)	616 (100)	901 (73)	1540 (50)	2060 (40)	3850 (25)	9750 (12)		

$(t/\rho) \times 100$ ratios in parentheses are for $P(A) = .95$ (or less)

Table 3c6

Sampling Plans for β = 1 2/3, r = .99

c	n $(t/\rho) \times 100$ Ratio for which P(A) = .10 (or less) 1000	800	500	300	250	200	150	100	80	50	40	25	15
0	5 (101)	8 (76)	16 (50)	37 (30)	50 (25)	73 (20)	117 (15)	230 (10)	334 (8.2)	731 (5.1)	1050 (4.1)	2300 (2.6)	5360 (1.5)
1	9 (237)	13 (187)	27 (117)	63 (70)	85 (59)	123 (47)	198 (35)	389 (23)	564 (19)	1240 (14)	1770 (9.5)	3890 (6.0)	9050 (3.7)
2	13 (316)	18 (255)	38 (160)	86 (95)	116 (81)	169 (64)	272 (48)	532 (32)	771 (26)	1690 (16)	2420 (13)	5320 (8.0)	
3	16 (385)	23 (303)	47 (193)	108 (115)	146 (95)	213 (76)	341 (57)	668 (38)	968 (30)	2120 (19)	3040 (15)	6680 (9.5)	
4	20 (420)	27 (345)	57 (215)	130 (129)	176 (106)	254 (85)	408 (64)	799 (43)	1158 (34)	2540 (21)	3630 (17)	8000 (11)	
5	23 (459)	32 (370)	66 (236)	150 (141)	205 (115)	295 (92)	473 (69)	928 (46)	1350 (37)	2950 (23)	4220 (19)	9280 (11)	
6	26 (490)	36 (395)	75 (249)	172 (148)	233 (123)	335 (98)	537 (74)	1060 (49)	1530 (39)	3340 (24)	4790 (20)		
7	29 (510)	41 (410)	84 (262)	193 (154)	260 (129)	375 (104)	600 (77)	1180 (51)	1710 (41)	3740 (26)	5350 (21)		
8	32 (535)	45 (430)	92 (272)	213 (160)	287 (134)	414 (107)	663 (81)	1300 (54)	1880 (43)	4130 (27)	5910 (22)		
9	36 (540)	49 (445)	101 (281)	233 (166)	314 (139)	452 (111)	725 (83)	1420 (56)	2060 (45)	4510 (28)	6460 (22)		
10	40 (540)	54 (445)	112 (283)	253 (171)	341 (143)	491 (115)	786 (86)	1540 (57)	2230 (46)	4890 (28)	7010 (23)		

$(t/\rho) \times 100$ ratios in parentheses are for P(A) = .95 (or more)

Table 3c7

Sampling Plans for $\beta = 2$, $r = .99$

c	n												
	$(t/\rho) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	800	500	400	300	250	200	150	120	100	80	50	40	25
0	4 (110)	10 (71)	15 (58)	26 (43)	37 (37)	58 (30)	102 (22)	156 (17)	230 (15)	349 (12)	903 (7.4)	1,400 (5.9)	3,600 (3.7)
1	7 (230)	16 (150)	25 (120)	44 (90)	63 (74)	98 (60)	169 (45)	263 (36)	389 (29)	589 (24)	1530 (15)	2,360 (12)	6,080 (7.5)
2	10 (300)	23 (190)	34 (150)	60 (110)	86 (97)	134 (77)	232 (58)	360 (47)	532 (39)	806 (31)	2,090 (19)	3,230 (16)	8,320 (10)
3	13 (340)	29 (220)	43 (180)	76 (130)	108 (110)	167 (90)	291 (67)	452 (54)	668 (45)	1,010 (36)	2,620 (22)	4,050 (18)	
4	15 (380)	34 (240)	52 (190)	91 (140)	130 (120)	200 (99)	348 (74)	540 (60)	799 (49)	1,210 (40)	3,140 (25)	4,850 (21)	
5	18 (410)	40 (260)	60 (210)	105 (160)	150 (130)	232 (100)	403 (79)	627 (64)	928 (52)	1,410 (43)	3,640 (26)	5,620 (21)	
6	20 (440)	45 (270)	69 (220)	120 (160)	170 (140)	263 (110)	458 (83)	712 (67)	1,050 (54)	1,600 (45)	4,130 (28)	6,380 (22)	
7	23 (450)	51 (280)	77 (230)	134 (170)	190 (140)	294 (110)	512 (87)	795 (70)	1,180 (58)	1,780 (47)	4,620 (29)	7,130 (23)	
8	25 (470)	57 (290)	85 (240)	148 (180)	210 (150)	325 (120)	565 (90)	878 (73)	1,300 (59)	1,970 (48)	5,090 (30)	7,870 (24)	
9	28 (475)	62 (300)	93 (240)	162 (182)	229 (153)	355 (122)	618 (92)	960 (74)	1420 (61)	2150 (50)	5570 (31)	8610 (25)	
10	32 (480)	69 (305)	103 (245)	175 (187)	249 (155)	385 (125)	670 (94)	1040 (76)	1540 (62)	2340 (51)	6040 (32)	9340 (25)	

$(t/\rho) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)

Table 3c8

Sampling Plans for $\beta = 2\ 1/2$, $r = .99$

c	n $(t/\rho) \times 100$ Ratio for which $P(A) = .10$ (or less)												
	500	400	300	250	200	150	120	100	80	65	50	40	25
0	5 (101)	8 (83)	15 (64)	24 (53)	41 (43)	84 (32)	146 (26)	230 (22)	397 (17)	668 (14)	1320 (11)	2300 (8.7)	7430 (5.4)
1	8 (185)	13 (150)	26 (114)	40 (95)	69 (76)	141 (57)	248 (46)	389 (38)	671 (31)	1130 (25)	2230 (19)	3890 (15)	
2	11 (230)	18 (185)	35 (141)	55 (118)	95 (93)	195 (70)	339 (56)	532 (47)	918 (38)	1540 (31)	3040 (23)	5320 (18)	
3	14 (260)	23 (208)	45 (156)	69 (132)	119 (105)	245 (78)	426 (63)	668 (53)	1150 (42)	1940 (34)	3820 (26)	6680 (21)	
4	17 (280)	27 (225)	53 (170)	83 (142)	143 (114)	293 (84)	509 (68)	799 (56)	1380 (46)	2320 (37)	4570 (28)	8000 (22)	
5	20 (295)	32 (238)	62 (178)	96 (150)	168 (119)	340 (89)	591 (71)	928 (60)	1600 (48)	2690 (39)	5300 (30)	9280 (24)	
6	22 (312)	36 (250)	71 (185)	110 (155)	190 (124)	386 (93)	671 (74)	1050 (62)	1820 (50)	3050 (41)	6020 (31)		
7	25 (320)	41 (255)	79 (193)	123 (160)	213 (127)	431 (96)	750 (77)	1180 (64)	2030 (52)	3410 (42)	6730 (32)		
8	28 (328)	45 (265)	87 (199)	135 (165)	235 (132)	476 (98)	827 (79)	1300 (66)	2240 (53)	3770 (43)	7420 (33)		
9	31 (334)	49 (271)	95 (203)	148 (168)	257 (135)	521 (101)	905 (80)	1420 (67)	2450 (54)	4120 (44)	8120 (34)		
10	34 (335)	54 (275)	105 (204)	163 (170)	279 (137)	564 (103)	982 (82)	1540 (69)	2660 (55)	4470 (45)	8810 (34)		

$(t/\rho) \times 100$ ratios in parentheses are for $P(A) = .95$ (or more)